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LIPPINCOTT'S
MENTAL ARITHMETIC
EMBRACING

THE PRINCIPLES OF ANALYSIS AND
INDUCTION

BY

J. MORGAN RAWLINS, A.M.

AUTHOR OF "LIPPINCOTT'S PRACTICAL ARITHMETIC" AND "LIPPINCOTT'S
ELEMENTARY ARITHMETIC"



PHILADELPHIA

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PREFACE.

“ARITHMETIC—queen of the terrestrial sciences” is an encomium by no means inapplicable to what is distinctively called the *mental system*. That a very thorough and philosophical knowledge of Arithmetic in its various aspects may be obtained by processes carried on by the mind, in total independence of the mechanical help of pen or pencil, no longer needs demonstration. Multitudes of men and women are ready, at a minute’s warning, to rise up and give emphatic testimony to the beneficence of the mental system as an educational force.

Docendo discimus. Not only those who are taught, but the teachers themselves who have adopted this system, and are faithfully applying it, are daily made conscious of its power as it develops within them the noble faculty of reason and fashions the invaluable habit of concentration. As the mind of the pupil unfolds, the mind of the teacher expands; both go on seeing clearly, and both escape the ditch into which the blind, leader and follower, inevitably fall.

In these modern times, Arithmetic is no longer considered, or certainly ought not to be, a mere compilation of mechanical rules and methods of operations that have no reasons for their existence that anybody is bound to respect. A change has come, we may say, and the intel-

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lectual study of Arithmetic upon the Inductive Method, first introduced by Warren Colburn—a distinction more significant and lasting than a monument of brass—has become one of the most effective agents in imparting a philosophical knowledge of Arithmetic, and of thus bestowing upon our youth a high and much-needed degree of mental discipline.

Inductive Method means method by induction, and the inductive plan of study is, therefore, not to be regarded as opposing or underrating a system of methods except as arbitrarily prescribed or dictated. It freely accepts the devices of art as most helpful aids to the acquirement of skill in the solution of arithmetical problems and to a complete mastery of the science; but it insists that those devices be inductively derived, philosophically established, and thoroughly *understood*. Prescribed rules and set modes are, in many parts of the subject, quite indispensable as stepping-stones to progress and to the higher knowledge that lies beyond. When Cæsar determined to cross the Rhine, he found no bridge prepared for him; but he crossed, nevertheless,—*on a bridge of his own invention*. A great excellence of the inductive mental system is that, while it confronts the pupil with difficulties, it has also prepared him to surmount them *by his own efforts*.

The greatest praise of any system of education is that it puts a youth in proper relations with his environment, and trains him to self-reliance, enterprise, self-helpfulness, and heroism. To such a system certainly belongs the science of Mental Arithmetic, and the work we now present we trust sets forth, not unworthily, the principles upon which the science is based.

The fundamental element of the intellectual treatment of the subject is Analysis. Analysis lays the foundation for Induction (Inference), and Induction supplies principles and methods. The philosophical comprehension of mathematical truths can be rightfully arrived at in no other way than by the steps of procedure indicated.

The analyses given throughout the book are meant to be suggestive merely, and under no circumstances to be rigidly adhered to. Each Analysis, or Solution, as it may be called, has been separated into two or more distinct parts; part 1 giving the process of the solution, and parts 2 and 3 the explanation of the process. This has been done, not only for the sake of clearness, but to save time, for assuredly the bright pupil need not be required at every step to explain assertion.

Our book is progressive: it starts with the simplest combinations and proceeds by easy grades to a point sufficiently high to show somewhat the potentialities of Arithmetic when philosophically taught.

In using the book, the teacher should begin at that chapter or lesson that best suits the advancement of the class, and then *make haste slowly*.

J. M. R.

MENTAL ARITHMETIC.

PRIMARY PROCESSES.

LESSON I.

Assuming that pupils who use this book are able to count and to perform the simple operations of Arithmetic, the following exercises on the fundamental processes have been prepared with a view to giving pupils greater facility in the use of numbers. The foundation for future progress is here to be laid, and the best judgment of the teacher and the earnest, persistent efforts of the pupil are requisite to a full measure of success.

The following are the forty-five simple combinations that can be formed from the nine digits:

1	2	3	4	5	6	7	8	9	2	3	4	5	6	7
<u>1</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>								
8	9	3	4	5	6	7	8	9	4	5	6	7	8	9
<u>2</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>4</u>	<u>4</u>	<u>4</u>	<u>4</u>	<u>4</u>						
5	6	7	8	9	6	7	8	9	7	8	9	8	9	9
<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>6</u>	<u>6</u>	<u>6</u>	<u>6</u>	<u>7</u>	<u>7</u>	<u>7</u>	<u>8</u>	<u>8</u>	<u>9</u>

TO THE TEACHER.—Pupils should be drilled on these simple combinations until they can without hesitation name the sums. Vary the exercise by asking for the difference between the numbers. Point to the combinations, ask for the sum, and while the pupils have the sum in mind ask them to subtract each number from that sum. It is quite easy to lead a pupil to see that if 7 and 8 are 15, 7 from 15 will leave 8 and 8 from 15 will leave 7. Do not lightly esteem these simple exercises,

LESSON II.

1. By the process of Addition, how may the following numbers be formed?

4, 7, 6, 3, 2, 5, 9, 8, 10, 12, 11, 13, 15, 14, 18, 16, 17.

Model Answer.—4 may be made up of 1, 1, 1, and 1; of 2 and 2; or of 3 and 1.

2. How many must be taken from :

8 to leave 4? 7 to leave 2? 9 to leave 5?

6 to leave 3? 5 to leave 2? 10 to leave 6?

14 to leave 8? 11 to leave 4? 12 to leave 3?

16 to leave 9? 18 to leave 7? 17 to leave 9?

13 to leave 6? 15 to leave 8? 16 to leave 4?

Model Answer.—4 and 4 are 8; therefore, 4 must be taken from 8 to leave 4.

3. State promptly how many are :

3 and 2.	5 and 3.	6 less 4.	7 less 3.
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9 less 6.	10 less 5.	8 and 3.	5 and 6.
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8 and 7.	14 less 7.	6 and 7.	13 less 6.
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5 and 8.	13 less 5.	2 and 9.	16 less 9.
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9 less 2.	7 and 9.	15 less 6.	9 and 6.
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6 and 4.	9 less 7.	12 less 7.	9 and 9.
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18 less 9.	17 less 8.	9 and 8.	17 less 9.
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4. How many are :

9 and 8 less 4?	5 and 4 and 3 less 5?
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7 and 9 less 6?	3 and 5 and 6 less 4?
-----------------	-----------------------

7 and 8 less 9?	4 and 6 and 3 less 9?
-----------------	-----------------------

6 and 6 less 5?	4 and 9 and 5 less 9?
-----------------	-----------------------

5 and 9 less 6?	2 and 7 and 7 less 8?
-----------------	-----------------------

5. Taking 3 from each of the following, how many will be left: 9, 11, 8, 7, 12, 10, 5, 6?
 6. How many must be added to 4 to make the following: 9, 5, 7, 8, 6?
 7. How many must be taken from 12 to leave the following: 5, 8, 7, 3, 6, 4, 9?
-

LESSON III.

1. Jack paid 5 cents for a sandwich and 5 cents for sugar plums. How many cents did he pay for both?

Solution.—He paid for the sandwich and sugar plums 5 cents and 5 cents, which are 10 cents.

2. Jane picked 5 white roses and 4 red ones. How many roses did she pick?
3. James paid 5 cents for his lunch and had 3 cents left. How many cents had he at first?
4. Hannah is 6 years old? How old will she be 4 years hence?
5. A man earned 4 dollars on Monday and 4 dollars on Tuesday. How much did he earn in the two days?
6. Virginia had 7 dollars and found a dollar. How many dollars had she then?
7. Victoria paid 5 dollars for shoes and had 6 dollars remaining. How many dollars had she at first?
8. There are 6 letters in one name and 7 in another. How many letters in both names?
9. Bessie spent 7 weeks at the seashore and 5 weeks in the city. How many weeks did she spend at both places?

10. Bruce rode 8 miles on his bicycle on Monday and 7 miles on Tuesday. How many miles did he ride in the two days?

11. How many roses did Walter give away, if he gave Ruth 7 and May 9?

12. Two boys went out to gather berries. If one picked 8 quarts and the other 9 quarts, how many did both pick?

LESSON IV.

Addition is the process of finding the sum of two or more numbers. The sign of Addition is the erect cross, +. It is used instead of *and*, and is read "plus."

1. A man earned 8 dollars and found 2 dollars. How many dollars had he then?

Solution.—He then had 8 dollars + 2 dollars, which are 10 dollars.

2. A boy paid 9 cents for a slate and 2 cents for a pencil. How many cents did both cost?

3. A hat cost 3 dollars and a coat 8 dollars. How much did both cost?

4. A girl had 9 cents and her father gave her 8 cents. How many cents had she then?

5. I had 9 dollars and earned 9 dollars more. How many dollars had I then?

6. A farmer sold 7 sheep to one man and 6 to another. How many sheep did he sell?

7. A student had 4 books and bought 7 more. How many books had he then?
8. If there are 8 pears on one tree and 7 on another, how many on both trees?
9. How many words did Margaret attempt to spell if she misspelled 3 and spelled 9 correctly?
10. How many digits in each of these numbers: Ten, eleven, twelve, thirteen, fourteen, fifteen?
11. What does the sign + mean? What does it require you to find?

LESSON V.

1. If 7 roses are on a bush and Irene cuts off 4, how many will be on the bush?

Solution.—There will be on the bush the difference between 7 roses and 4 roses, which is 3 roses.

2. James had 12 marbles and lost 5. How many did he then have?
3. If John has 15 cents and Harry has 8 cents, how much more has John than Harry?
4. If a boy has 4 cents, how many must he get that he may have 12 cents?
5. Percy had 13 apples and gave his sister 5. How many did he then have?
6. Eight of the 17 apples that were on a tree dropped off. How many remained on the tree?
7. If Harry had 9 cents more he would have 17 cents. How many cents has Harry?

8. In two boxes there are 14 books, and in one of the boxes there are 6 books. How many are in the other?

Rapid work for blackboard drill:

$$\begin{array}{cccccccccccc} 7 & 8 & 9 & 6 & 7 & 8 & 9 & 6 & 5 & 4 & 5 \\ \underline{4} & \underline{6} & \underline{7} & \underline{4} & \underline{2} & \underline{7} & \underline{8} & \underline{7} & \underline{8} & \underline{5} & \underline{7} \end{array}$$

TO THE TEACHER.—The teacher will place these and similar combinations on the blackboard; then, pointing to each combination, say "add." The pupil will name *promptly* the sum of the combination. The teacher will then point to the first figure in the combination and say "subtract." Then point to the second figure. Drill pupils on these and similar combinations until pupils give results unhesitatingly.

LESSON VI.

1. Name promptly the sums as indicated:

$$\begin{array}{lllll} 4 + 3. & 3 + 6. & 7 + 4. & 7 + 6. & 8 + 7. & 9 + 8. \\ 2 + 5. & 5 + 4. & 8 + 3. & 8 + 5. & 9 + 6. & 7 + 9. \\ 4 + 4. & 6 + 4. & 6 + 6. & 7 + 7. & 8 + 8. & 4 + 8. \\ 3 + 5. & 3 + 7. & 7 + 5. & 7 + 8. & 9 + 9. & 9 + 3. \end{array}$$

2. Give results quickly:

$$\begin{array}{llll} 9 \text{ less } 3. & 11 \text{ less } 6. & 14 \text{ less } 6. & 15 \text{ less } 9. \\ 7 \text{ plus } 4. & 8 \text{ plus } 6. & 8 \text{ plus } 7. & 17 \text{ less } 8. \\ 10 \text{ less } 5. & 12 \text{ less } 7. & 15 \text{ less } 8. & 9 \text{ plus } 4. \\ 9 \text{ plus } 8. & 18 \text{ less } 9. & 9 \text{ plus } 6. & 16 \text{ less } 9. \\ 5 \text{ plus } 5. & 6 \text{ plus } 6. & 7 \text{ plus } 7. & 8 \text{ plus } 8. \end{array}$$

3. Combination exercises:

NOTE.—The teacher will read slowly and distinctly, and then name a member of the class to give the result.

8 plus 7 less 3.	12 less 9 plus 7.	17 less 8 plus 6.
6 plus 7 less 4.	18 less 9 plus 6.	7 plus 9 less 8.
9 less 3 plus 7.	8 plus 8 less 9.	6 plus 9 less 6.
11 less 4 plus 7.	16 less 7 plus 4.	13 less 6 plus 8.
5 plus 6 less 4.	8 plus 6 less 9.	10 less 4 plus 7.
4 plus 9 less 8.	15 less 9 plus 6.	3 plus 8 less 4.
6 less 6 plus 6.	7 plus 7 less 7.	8 less 6 plus 4.

4. Combination exercises for rapid work:

4 plus 3 less 2 plus 5.	14 less 9 plus 5 less 3.
3 plus 5 plus 6 less 7.	13 less 5 less 4 plus 6.
8 less 5 plus 4 plus 6.	6 plus 6 less 5 plus 5.
5 plus 8 less 6 plus 4.	16 less 9 plus 7 less 6.
7 plus 9 less 8 less 3.	17 less 8 plus 6 less 8.
5 plus 5 plus 5 less 8.	6 plus 6 plus 6 less 9.

LESSON VII.

The **Sign of Equality** is two parallel lines, $=$. It is read "equals," or "is equal to." $3 + 2 = 5$, is read "Three plus two equals five."

1. A merchant sold 7 barrels of flour to one man, and 5 to another man. How many barrels did he sell to both?

Solution.—7 barrels + 5 barrels = 12 barrels. Therefore, he sold 12 barrels.

2. I paid 9 cents for a book, and 4 cents for a pencil. How much did I pay for both?

3. A farmer paid 9 dollars for a plough and 8 dollars for a harrow. How much did he pay for both?

4. A baker bought 8 barrels of flour from one miller and 8 more from another. How many barrels from both?
 5. Frederick gave 6 cents for a purse, and had 7 cents left to put into it. How many cents had he at first?
 6. A farmer sold 5 cows, and then had 6 cows left. How many cows had he at first?
 7. If you receive 9 dollars from one man and 5 from another, how many dollars do you receive?
 8. A class contains 7 studious boys and 5 idlers. How many boys in the class?
 9. A class consists of 6 small boys and 5 large boys. How many boys in the class?
 10. In a field 8 sheep are standing up and 6 lying down. How many sheep in all?
 11. Mary is 7 years old and her brother is 2 years older. How old is her brother?
-

LESSON VIII.

Subtraction is the process of finding the difference between two like numbers.

The **Sign of Subtraction** is a short horizontal line, —. It is called "minus," and when placed between two numbers it means that the number following it is to be taken from that which precedes it. The expression $8 - 5 = 3$, is read "Eight minus five equals three."

1. If Emily has 13 cents and Harry has 7 cents, how many more cents has Emily than Harry?

Solution.—13 cents — 7 cents = 6 cents. Therefore, Emily has 6 cents more than Harry.

2. Oscar wants to buy a ball for 15 cents, but he has only 9 cents. How many cents does he need?

3. Julia bought 7 cents' worth of paper and gave the shopkeeper 10 cents. How much change did she get?

4. If 16 birds are sitting on the fence and 7 fly away, how many remain?

5. John is 14 miles from home. After walking 6 miles, how far has he to walk to reach home?

6. Roy had 11 marbles and lost 4. How many did he then have?

7. Anna spent 18 cents for a book and 9 cents for a slate. How much less did she pay for the slate than for the book?

8. Harry was given 12 words to spell. If he spelled 8 correctly, how many did he miss?

9. A boy having 17 cents spent 8 cents for peaches. How many cents had he left?

LESSON IX.

1. Louis had 9 marbles and lost 4 of them. How many had he left?

Solution.—He had left 9 marbles — 4 marbles, or 5 marbles.

2. John has 10 cents and Mary has 6. How many more has John than Mary?

3. If there are 12 boys belonging to a class and 8 are present, how many are absent?

4. If there are 11 crows on a tree and 6 fly away, how many will be left?

5. Bertha had 13 cents and spent 7 cents. How much had she left?

6. I sold some wheat for 15 dollars which cost me 6 dollars. What was my gain?

7. Mary is now 16 years old. How old was she 8 years ago?

8. Mr. A. having 12 sheep sold 9. How many had he remaining?

9. Ann has 12 books and Julia has 7. How many more must Julia have to make her number equal Ann's.

10. Of 17 sheep which were in a fold 9 escaped; how many remained in the fold?

11. A hunter shot 16 rabbits and 15 partridges on Monday, and on Tuesday he shot 7 rabbits and 8 partridges. How many more of each did he shoot on Monday than on Tuesday?

12. Exercise in rapid addition and subtraction:

$$1. 9 + 8 - 6 + 9 - 3 - 4 + 6 - 10 + 8 = ?$$

$$2. 16 - 8 + 4 - 6 + 3 + 8 - 12 + 8 + 7 = ?$$

$$3. 19 - 12 + 7 - 14 + 6 - 3 + 17 - 10 + 5 + 1 = ?$$

$$4. 1 + 2 + 3 + 4 - 5 + 6 + 7 + 2 - 10 + 3 - 8 = ?$$

$$5. 20 - 2 = 3 - 4 - 5 + 17 - 9 + 8 - 2 = ?$$

$$6. 5 + 5 + 5 - 2 - 2 - 2 + 9 - 3 - 3 - 3 - 1 = ?$$

$$7. 17 - 9 + 6 - 5 - 1 + 7 - 3 + 4 - 7 + 9 = ?$$

$$8. 7 + 7 + 3 + 2 - 1 - 2 - 3 - 4 + 6 - 4 + 2 = ?$$

To THE TEACHER.—Read this exercise slowly, have the whole class work silently, and require pupils to give results individually. Similar exercises should be given frequently.

LESSON X.

Miscellaneous Exercises.

Tell the sums of the following columns:

SUGGESTION.—The teacher will read, simply naming the numbers.
Pupils should answer promptly.

1.

(A.)	(B.)	(C.)	(D.)	(E.)	(F.)	(G.)	(H.)	(I.)	(J.)
4	4	3	8	9	8	6	7	9	3
3	7	7	4	3	6	5	6	6	7
<u>7</u>	<u>8</u>	<u>6</u>	<u>3</u>	<u>6</u>	<u>4</u>	<u>6</u>	<u>7</u>	<u>4</u>	<u>5</u>

2.

(A.)	(B.)	(C.)	(D.)	(E.)	(F.)	(G.)	(H.)	(I.)	(J.)
3	2	9	8	7	6	8	3	8	7
5	4	3	4	8	9	4	4	9	8
<u>7</u>	<u>6</u>	<u>6</u>	<u>5</u>	<u>5</u>	<u>9</u>	<u>6</u>	<u>4</u>	<u>8</u>	<u>7</u>

3.

(A.)	(B.)	(C.)	(D.)	(E.)	(F.)	(G.)	(H.)	(I.)	(J.)
6	7	4	4	7	8	8	3	8	5
4	3	9	6	6	6	8	8	4	4
<u>9</u>	<u>7</u>	<u>6</u>	<u>6</u>	<u>2</u>	<u>7</u>	<u>6</u>	<u>2</u>	<u>7</u>	<u>4</u>

4.

(A.)	(B.)	(C.)	(D.)	(E.)	(F.)	(G.)	(H.)	(I.)	(J.)
2	3	6	2	7	3	4	7	5	7
6	4	7	5	6	6	8	8	8	6
<u>8</u>	<u>5</u>	<u>4</u>	<u>6</u>	<u>8</u>	<u>7</u>	<u>4</u>	<u>9</u>	<u>1</u>	<u>4</u>

SUGGESTION.—Have pupils add both silently and audibly, naming only results. While the pupil has the sum of a column in mind, have him subtract therefrom each number separately. This exercise is susceptible of diversity and will enlist attention.

LESSON XI.

Combinations of Four Numbers.

1.

(A.)	(B.)	(C.)	(D.)	(E.)	(F.)	(G.)	(H.)	(I.)	(J.)
3	1	4	4	9	8	3	5	2	4
4	3	3	7	8	2	7	8	3	6
4	3	7	8	8	4	6	8	4	3
7	9	6	9	6	6	5	7	6	8

2.

(A.)	(B.)	(C.)	(D.)	(E.)	(F.)	(G.)	(H.)	(I.)	(J.)
5	8	7	8	6	9	6	8	5	2
7	8	8	8	9	5	6	6	7	5
7	6	6	7	6	6	2	8	8	8
8	6	4	9	4	9	4	8	8	4

3.

(A.)	(B.)	(C.)	(D.)	(E.)	(F.)	(G.)	(H.)	(I.)	(J.)
4	3	7	3	8	8	7	4	4	8
9	4	8	4	8	5	8	7	7	4
6	5	9	6	4	7	4	5	6	8
9	8	7	8	6	3	9	8	7	5

4.

(A.)	(B.)	(C.)	(D.)	(E.)	(F.)	(G.)	(H.)	(I.)	(J.)
9	2	8	5	2	3	4	4	3	7
9	6	8	7	4	3	8	6	4	7
5	6	8	8	5	6	9	9	5	8
6	4	6	8	8	8	9	9	8	9

LESSON XII.

1. Add 3 to each of these : 6, 7, 5, 8, 3, 4, 5, 6, 7, 9, 2, 8, 6, 4.
2. Add 5 to each of these : 3, 4, 7, 5, 8, 6, 2, 9, 6, 8, 4, 3, 4, 3.
3. Add 7 to each of these : 5, 4, 8, 6, 7, 4, 5, 2, 3, 9, 6, 5, 3, 6.
4. Add 9 to each of these : 2, 3, 4, 6, 5, 2, 8, 3, 6, 7, 4, 9, 3, 8.
5. Add 8 to each of these : 3, 2, 9, 5, 3, 6, 7, 8, 3, 9, 7, 4, 2, 5.
6. Count by 2's from 0 to 20 ; by 3's to 21 ; by 4's to 20.
7. Subtract 5 from 13, 5, 9, 8, 12, 5, 7, 9, 6, 8, 15, 12, 10, 11, 18, 19, 17, 14.
8. Subtract 3 from 6, 9, 7, 5, 12, 8, 9, 4, 13, 17, 19, 18, 11, 10, 16, 15, 8, 14.
9. Subtract 7 from 20, 7, 17, 13, 18, 14, 11, 9, 10, 8, 12, 16, 15, 7, 8, 9, 14, 19.
10. Subtract 9 from 11, 17, 13, 10, 15, 18, 21, 19, 12, 16, 14, 20, 9, 22.
11. Subtract 6 from 15, 11, 13, 8, 12, 17, 14, 10, 19, 20, 18, 9, 21, 15, 16.
12. Subtract 8 from 10, 13, 17, 11, 9, 19, 12, 18, 16, 15, 8, 14, 19, 20, 21.
13. Subtract 9 from 9, 11, 13, 15, 17, 18, 19, 10, 12, 14, 16, 18, 20, 22, 21.
14. Count by 6's from 3 to 100 ; by 7's from 1 to 101.
15. Subtract by 3's from 100 to 1.

LESSON XIII.

1. How many figures are used to represent numbers ?

SUGGESTION.—Place correct forms on the blackboard.

2. Which figure represents no value ?
3. What is the largest number that can be expressed with one figure ?

4. Tell how the following numbers are written :

Eleven.	Ten.	Twelve.
Seventeen.	Fifteen.	Nineteen.
Twenty.	Twenty-one.	Twenty-three.
Twenty-seven.	Twenty-five.	Twenty-six.
Thirty.	Thirty-seven.	Forty.
Fifty-eight.	Fifty-five.	Sixty.
Seventy-three.	Seventy-five.	Eighty.
Ninety-seven.	Ninety-four.	Ninety-nine.

5. What is the largest number that can be expressed with two figures ?

6. Analyze the numbers in No. 10, Lesson 12, by stating how many units and tens in each.

7. Between 1 and 111 there are ten numbers ending with 5 ; name these numbers and add 6 to each.

8. Between 1 and 111 there are ten numbers ending with 6 ; name them and add 7 to each.

9. Name the ten numbers within the same limits which end with 7, and add 8 to each.

10. Name those ending with 8, and add 9 to each.

11. Name those ending with 4, and add 8 to each.

NOTE.—The teacher should always endeavor to grasp the purpose which the author had in mind when making the lesson, and then so conduct the recitation as to accomplish that purpose.

LESSON XIV.

1. Name the numbers between 1 and 111 which end with 1 and subtract 2 from each.
2. Name those ending with 2 and take 3 from each.
3. Name those ending with 4 and take 5 from each.
4. Name those ending with 3 and take 6 from each.
5. Name those ending with 0 and take 4 from each.
6. Name those ending with 5 and take 6 from each.
7. Name those ending with 3 and take 7 from each.
8. Name those ending with 4 and take 9 from each.
9. Name those ending with 6 and take 8 from each.
10. Name those ending with 7 and take 9 from each.
11. Name those ending with 8 and take 9 from each.
12. Name those ending with 2 and take 7 from each.

NOTE.—The purpose of the foregoing exercise is obvious, and if the exercise is not sufficient to accomplish that purpose the teacher should extend it.

13. Give answers at sight:

(A.)	(B.)	(C.)	(D.)	(E.)
10 — 1.	10 — 9.	15 — 8.	19 — 9.	28 — 5.
12 — 1.	11 — 8.	14 — 7.	28 — 9.	49 — 6.
15 — 3.	12 — 7.	16 — 5.	27 — 9.	77 — 7.
15 — 4.	13 — 6.	13 — 6.	36 — 9.	35 — 8.
13 — 5.	14 — 5.	17 — 4.	45 — 9.	46 — 9.
14 — 3.	15 — 4.	12 — 3.	54 — 9.	13 — 0.
16 — 6.	16 — 3.	18 — 2.	33 — 9.	64 — 6.
17 — 8.	17 — 2.	11 — 4.	72 — 9.	51 — 3.
18 — 9.	18 — 1.	19 — 0.	81 — 9.	12 — 2.
11 — 10.	19 — 0.	10 — 4.	90 — 9.	70 — 7.

LESSON XV.

United States Money.

1. \$10 means 10 dollars. What does \$75 mean?
 2. To separate dollars and cents a mark called the Decimal Point (.) is used. Six dollars and twenty-five cents is written \$6.25; four dollars and eight cents is written \$4.08. When the decimal point is used cents are invariably expressed with two figures.
 3. $\$10 + \$5 = \$15$, is an expression of equality, and is called an **Equation**.
 4. Form an equation with \$20 and \$5.
- Solution.**— $\$20 + \$5 = \$25$.
5. Form an equation with the following sums of money:

\$10 and \$2.	\$20 and \$5.	\$45 and \$10.	\$85 and \$10.
\$3 and \$11.	\$25 and \$5.	\$55 and \$5.	\$95 and \$5.
\$12 and \$6.	\$30 and \$5.	\$60 and \$10.	\$90 and \$10.
\$7 and \$13.	\$35 and \$5.	\$70 and \$5.	\$80 and \$5.
\$8 and \$12.	\$40 and \$5.	\$80 and \$10.	\$85 and \$5.
 6. How many cents in \$1? \$1 = how many cents?
 7. How many cents in 1 dime? How many dimes in \$1.
 8. Put each answer in the form of an equation.
 9. My hat cost 50 cents and my top 10 cents. What did both cost?
 10. George paid 75 cents for a ball and 10 cents for a bat. What did both cost?
 11. A boy by selling a bat for \$.20 lost \$.10. How much did it cost him?

LESSON XVI.

SUGGESTION.—To find the sum of large numbers, take one of the numbers, add to it the units of another number, and then add the tens of that number, and so on. Suppose it be required to find the sum of 49 and 56. Starting with 49, add the 6 units of 56, and then the 5 tens. Proceed in like manner with the other numbers to be added.

1. Apply this method to the following, adding by columns:

12	11	17	15	16	26	14	14
14	12	11	18	12	27	16	15
10	13	16	22	13	13	30	17
<u>16</u>	<u>14</u>	<u>24</u>	<u>25</u>	<u>20</u>	<u>15</u>	<u>42</u>	<u>18</u>

2. In like manner find the sums of the following:

$$\begin{array}{llll} 37 + 29. & 43 + 37. & 76 + 46. & 29 + 48. \\ 69 + 38. & 87 + 49. & 94 + 37. & 58 + 63. \\ 83 + 67. & 44 + 57. & 89 + 64. & 79 + 98. \\ \underline{54} + \underline{98}. & \underline{39} + \underline{56}. & \underline{77} + \underline{66}. & \underline{47} + \underline{87}. \end{array}$$

3. Add the vertical columns in No. 2.

4. Add the following numbers by starting with one number and adding to it the tens of the next number and then the units, and so on:

43	27	45	24	41	19	37	97	39
64	33	98	45	76	39	47	43	48
<u>85</u>	<u>67</u>	<u>76</u>	<u>82</u>	<u>64</u>	<u>59</u>	<u>57</u>	<u>64</u>	<u>94</u>

To THE TEACHER.—Give pupils an abundance of similar exercises, and apply this method to all written work in addition. Do not permit pupils to name the figures; have them name only the result at each successive step. Use the blackboard.

LESSON XVII.

1. If I owe you 81 cents and you owe me 37 cents, how can we justly make settlement?
2. Charles has a certain number of chestnuts. If he had 59 more he would have 200. How many has he?
3. If you sell an article for 66 cents and thus gain 16 cents, what did the article cost you?
4. New York and Philadelphia are 90 miles apart. When you have travelled 43 miles of the distance, how many miles remain to be travelled?
5. If you buy for \$33 and sell for \$42, how much do you gain?
6. If you sell for \$66 and gain \$16, what was the cost?
7. Take \$54 from \$82. \$58 from \$100.
8. Subtract \$.28 from \$.54. \$.87 from \$1.24.
9. General Grant lived 63 years. President Lincoln lived 56 years. How much older was Grant at death than Lincoln?
10. How many fowls did a fox catch if he left only 43 out of 81?
11. After spending \$38 and earning \$16, a boy had \$46 lacking \$17. How many dollars had he at first?
12. A mechanic earns \$5.50 Monday, \$6.50 Tuesday, \$7.50 Wednesday, \$8.50 Thursday, \$4.50 Friday, \$5.50 Saturday. How much does he earn in a week?
13. Tell the sum of:

(1)	(2)	(3)	(4)	(5)
\$3.25	75.50	36.75	\$62.50	\$3.25
\$6.00	6.25	27.00	\$7.50	\$32.75
<u>\$87.50</u>	<u>3.50</u>	<u>6.50</u>	<u>\$16.25</u>	<u>\$7.25</u>

LESSON XVIII.

1. Allen rode his bicycle 22 miles on Monday, 18 miles on Tuesday, and 50 miles on Wednesday. How many miles did he ride in the three days?

2. A merchant sold some rice for \$25, some sugar for \$20, and some syrup for \$32. How many dollars did he receive for all?

3. A mechanic paid \$31 for boards, \$10 for shingles, \$6 for nails, and \$5 for screws. How many dollars did he spend?

4. On July 4th a boy spent 6 cents for fruit, 12 cents for confections, 12 cents for a United States flag, gave away 5 cents, and had 7 cents left. How much money had he at first?

5. A man bought a cow and a pig. The pig cost \$11. He paid \$42 more for the cow than for the pig. What did the cow cost him?

6. There are 18 boys in the first form, 20 in the second, 14 in the third, and 17 in the fourth. How many boys in the four forms?

7. How many days in March, April, and May?

8. I have a triangular piece of ground. The longest side is 35 yards; the other two sides are 25 and 19 yards respectively. How many yards around my lot?

9. If John pays 15 cents for a top, 75 cents for a knife, and 87 cents for a book, what does he pay for all?

10. A gentleman bought 3 bicycles. He paid \$50 for one, \$65 for the second, and \$33 for the third. How much did all cost him?

11. A farmer sold 36 sheep, lost 17, and had 73 left. How many had he at first?

LESSON XIX.

SUGGESTION.—When numbers are large, Subtraction can be most easily performed by first subtracting the tens of the subtrahend and then the units. $82 - 17 =$ what? $82 - 1$ ten $= 22$; $22 - 7 = 15$.

1. Apply this suggestion to the following:

$$\begin{array}{llll} 32 - 17. & 98 - 59. & 62 - 47. & 77 - 69. \\ 46 - 29. & 76 - 48. & 87 - 39. & 85 - 58. \\ 58 - 19. & 93 - 37. & 91 - 47. & 67 - 19. \\ \underline{64 - 37.} & \underline{84 - 36.} & \underline{57 - 29.} & \underline{52 - 27.} \end{array}$$

2. Add the columns in No. 1.
 3. Subtract 41 from 67; 19 from 41; 41 from 95.
 4. Find the difference between 55 and 93; 26 and 73.
 5. Take 44 from 71; 33 from 81; 22 from 91.
 6. How much is 28 less than 45? 38 less than 55?
 7. Subtract 66 from 110; 58 from 120; 57 from 130.
 8. Answer the following inquiries:

SUGGESTION.—Subtract hundreds first, then tens, and lastly units.

$$\begin{array}{lll} 112 - 37 = ? & 134 - 49 = ? & 131 - 34 = ? \\ 120 - 64 = ? & 123 - 66 = ? & 123 - 86 = ? \\ 155 - 75 = ? & 175 - 91 = ? & 114 - 25 = ? \\ 157 - 93 = ? & 176 - 75 = ? & 110 - 91 = ? \\ 161 - 85 = ? & 184 - 55 = ? & 130 - 81 = ? \\ 400 - 220 = ? & 480 - 170 = ? & 300 - 190 = ? \\ 427 - 127 = ? & 400 - 190 = ? & 500 - 296 = ? \\ 475 - 250 = ? & 488 - 370 = ? & 880 - 585 = ? \end{array}$$

NOTE.—Use the blackboard at first, but do not allow pupils to write results. Dispense with the use of slate, paper, and blackboard as soon as possible, and thus make the exercises purely mental.

LESSON XX.

Rapid Work in Addition and Subtraction.

1. Find the value of the following:

- | | | |
|--------------------|---------------------|---------------------|
| 1. $8 + 27 - 10.$ | 6. $9 + 14 - 10.$ | 11. $24 + 19 - 30.$ |
| 2. $13 - 6 + 9.$ | 7. $15 - 8 + 7.$ | 12. $18 - 12 + 28.$ |
| 3. $14 - 10 + 13.$ | 8. $33 - 21 + 10.$ | 13. $42 - 20 - 10.$ |
| 4. $17 + 13 - 11.$ | 9. $60 - 35 + 7.$ | 14. $16 + 19 - 15.$ |
| 5. $15 + 10 - 13.$ | 10. $19 + 18 - 30.$ | 15. $12 + 18 - 14.$ |

2. Complete the following equations:

- | | |
|-----------------------|-----------------------------|
| 1. $18 - 9 + 33 = ?$ | 6. $13 + 9 + 7 - 7 + 4 = ?$ |
| 2. $20 + 18 - 21 = ?$ | 7. $19 + 18 - 13 - 9 = ?$ |
| 3. $15 + 17 - 13 = ?$ | 8. $33 + 35 - 30 - 28 = ?$ |
| 4. $17 + 51 - 27 = ?$ | 9. $35 + 85 - 46 - 34 = ?$ |
| 5. $33 + 28 - 31 = ?$ | 10. $45 + 75 - 35 + 30 = ?$ |

3. A man travelled 8 miles before breakfast, 30 miles more before midday, and 40 more after midday. How much did he lack of travelling 100 miles?

Solution.—1. The process is $8 + 30 + 40 = 78$; $100 - 78 = 22$.

2. He lacked the difference between 100 miles and the sum of 8 miles, 30 miles, and 40 miles.

3. Therefore, he lacked 22 miles of travelling 100 miles.

4. A piece of ground cost \$13 for improvement and \$36 for cultivation. Its products amounted to \$48. How much did the owner lose?

5. A farmer sheared 500 sheep in 2 weeks. The first week he sheared 270. How many did he shear the second week?

6. Mr. Smith owes his butcher, grocer, and baker \$365. He owes his grocer \$220. How much does he owe the other two?
7. From 525 subtract $200 + 5$.
8. I owed a man \$17 and gave him in part payment \$8 worth of corn. How much remained due him?
9. A man bought sugar for 9 dollars and sold it for \$15. How much did he gain?
10. A girl lives 12 blocks away. After she has passed 7 blocks, how many more must she pass before reaching home?
11. From the sum of 9 and 5 subtract 10.
12. What remains of \$3.75 after \$1.64 has been spent?
13. I have read 40 pages in a book which contains 90 pages. How many pages remain to be read?
14. Four years ago Dickinson left school. He is now 19. How old was he then?
15. A farmer bought a cow for \$20 and sold her for \$30. How much did he gain?
16. A man earns \$25 a week and spends \$7 a week for board. How much does he save each week?
17. A school has 29 pupils. 19 are girls. How many are boys?
18. If an ox costing \$36 was sold for \$50, what was the gain?
19. If in a school of 45 pupils 15 are promoted, how many failed of promotion?
20. I had a certain number of sheep, bought 7, sold 11, and then had none left. How many had I at first? .

LESSON XXI.

1. A boy paid 5 dimes for a knife, 10 dimes for a book, and 15 dimes for a hat. How many dimes did he pay for all?

Solution.—5 dimes + 10 dimes + 15 dimes = 30 dimes.
Therefore, he paid for all 30 dimes.

2. Henry had 29 cents and earned 10 cents more. How much had he then?

3. I paid 50 cents for apples, 40 cents for peaches, and 10 cents for pears. How much did I pay for all?

4. A horse cost \$75 and a carriage \$25 more than the horse. What did both cost?

5. A grain dealer paid \$85 for wheat, \$15 for corn, and \$15 for oats. How much did he pay for all?

6. A carriage cost \$30 more than a cart. How much did both cost if \$38 was paid for the cart?

7. A wagon cost me \$85; the repairing of it, \$14. I then sold it at a profit of \$11. Find the selling price.

8. On Monday a depositor had in bank a balance of \$55. On Tuesday his balance was \$45 larger. What was his total balance on Tuesday?

9. A desk cost \$35, a bureau \$20 more, and a mirror as much as both the desk and bureau. Find the cost of all.

10. From 106 take the sum of 55 and 44.

11. A man earned \$150 per month. He paid \$25 for rent and \$70 for other expenses. He saved monthly how much?

12. Cotton was first planted in the United States about 1759. How long ago?

13. A man gave his oldest son \$400, his second son \$125 less, and his third son \$25 less than the second. How much did all receive?

14. A man dies worth \$1200. He leaves \$800 to his daughter, and the remainder to his son. What did the son receive?

15. A merchant bought 50 barrels of flour for \$500. He sold 25 barrels for \$200. How many barrels remained, and how much must he sell them for that he may lose nothing?

LESSON XXII.

1. Mr. A. started out to walk 40 miles, but after he had walked 29 miles he stopped. He had to walk how many more miles to complete the distance?

2. When 17 is subtracted from 46, how many tens and how many units remain?

3. A horse and cow cost together \$276. If the cow cost \$61, what did the horse cost?

4. If a man bought a horse for \$125 and sold him for \$185, what was his gain?

5. A farmer had 378 sheep and sold 256. How many had he left?

6. I bought a horse for \$185 and sold it for \$212. How much did I gain?

7. A drover bought some sheep for \$40 and some cattle for \$130; he then sold them all for \$200. How much did he gain?

8. I bought a watch for \$100, a chain for \$37, and sold both for \$92. How much did I lose?

9. A boy added two numbers and his answer was 70. If one of the numbers was 37, what was the other?
10. I bought a horse for \$100, paid \$25 for the care of him, and sold him for \$170. What was my gain?
11. The sum of the ages of A., B., and C. is 146 years. A. is 46 years, and B. is 10 years older. Find the age of C.
12. A cable 100 feet long was cut into three pieces. One piece was 53 feet long and another was 13 feet shorter. What was the length of the third piece?
13. A farmer has 535 sheep, 40 cows, 26 horses, 30 calves, and 10 colts. If he sells the sheep, how many animals will he have left?
14. In a garden are 68 fruit trees; 25 of them are peach trees, 13 plum trees, and the remainder pear trees. How many pear trees are there?
15. If a man owes \$87.55 and pays \$50.50, how much does he still owe?
16. A grocer bought a barrel of apples, containing 457 apples. He sold 215 apples. How many apples were left?
17. A grocer's bill of purchase was as follows: Oranges, \$18; lemons, \$8.00; raisins, \$4.00; figs, \$6.00. He sold the whole for \$44. Did he gain or lose, and how much?
18. A man has in bank \$77, deposits \$88, and draws out \$66. How much remains in bank?
19. Sixty sheep are in one fold, 48 in a second, and 35 in a third. If 11 escape from the first, 6 from the second, and 3 from the third, how many will remain?
20. A boy paid \$5.75 for a coat and a pair of shoes; for the shoes he paid \$2.22. How much did he pay for the coat?

LESSON XXIII.

In an equation, that which precedes the sign of equality is called the **First Member**; that which follows the sign of equality is the **Second Member**.

Find the second member of the following equations:

1. $36 + 16 - 20 + 24 - 6 = ?$
2. $57 + 24 - 40 + 19 - 20 = ?$
3. $48 - 17 + 49 - 23 - 12 = ?$
4. $68 + 24 - 37 + 42 - 34 = ?$
5. $97 + 28 + 43 - 90 - 27 = ?$
6. $39 + 49 - 16 + 31 - 11 = ?$
7. $52 - 27 + 25 - 20 + 47 = ?$
8. $64 - 37 + 56 - 31 - 19 = ?$
9. $19 + 17 - 16 + 21 - 14 = ?$
10. $76 + 56 - 50 + 40 - 49 = ?$
11. $36 - 26 + 33 + 8 - 28 + 3 - 4 + 19 - 13 = ?$
12. $19 + 29 - 9 + 35 - 16 + 13 - 11 + 50 = ?$

Blackboard Exercise.

Place upon the blackboard long columns of numbers, commencing with two-figure numbers, and gradually increase by adding hundreds, thousands, etc., and then, pointing to the figures, have pupils add both silently and audibly, as suggested in Lesson XVI. Such exercise is valuable, and will lead pupils to apply the method to their written work. Do not attempt to add more than two columns at a time.

SUGGESTION.—Do not pass to the next subject unless pupils are able to perform easily, rapidly, and correctly the processes of Addition and Subtraction. Review the lessons gone over.

LESSON XXIV.

Multiplication.

The sign of Multiplication is the oblique cross, \times .

4×5 is read "four multiplied by 5," or "four times 5."

Multiplication is a short method of **Addition**.

The result of Multiplication is called the **Product**.

The numbers multiplied together are called **Factors** of the product.

1. What will 4 apples cost at 3 cents apiece ?

Solution.—Since 1 apple costs 3 cents, the 4 apples will cost 3 cts. + 3 cts. + 3 cts. + 3 cts. = 12 cents. Therefore, the 4 apples will cost 4 times 3 cents, or 12 cents.

2. How much will 4 oranges cost at 5 cents apiece ?

3. How much will 3 lemons cost at 3 cents apiece ?

4. How much will 5 bananas cost at 3 cents each ?

5. A man gave 10 cents to each of 5 poor children.

How much did he give to all ?

6. If there are 3 feet in one yard, how many feet are there in 10 yards ?

7. There are 7 days in one week. How many days are there in 3 weeks ?

8. Find the value of 6 yards of calico at 5 cents a yard ?

9. There are 8 quarts in a peck. How many quarts in 5 pecks ?

10. If a horse eats 8 quarts of oats per day, how many quarts does he eat in 7 days ?

11. If a man earns \$9 in one week, how many dollars can he earn in 6 weeks ?

12. If a horse travels 8 miles per hour, how far will he travel in 9 hours ?

LESSON XXV.

The result of multiplying one number by another is called the **Product**.

Tell the products of the following numbers:

$$\begin{array}{r} & & & & & \text{1.} \\ \begin{array}{rrrrrrrrrr} 6 & 9 & 5 & 4 & 2 & 8 & 6 & 7 & 8 \\ -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 & -2 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} & & & & & \text{2.} \\ \begin{array}{rrrrrrrrrr} 11 & 10 & 12 & 3 & 8 & 7 & 6 & 5 & 9 \\ -4 & -4 & -4 & -4 & -4 & -4 & -4 & -4 & -4 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} & & & & & \text{3.} \\ \begin{array}{rrrrrrrrrr} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ -6 & -6 & -6 & -6 & -6 & -6 & -6 & -6 & -6 \end{array} \\ \hline \end{array}$$

SUGGESTION.—When teaching these combinations ask pupils to tell how many times each factor is contained in the product. A pupil can readily see that 6 times 8 being 48, 8 is contained 6 times in 48, and 6 is contained 8 times in 48.

$$\begin{array}{r} & & & & & \text{4.} \\ \begin{array}{rrrrrrrrrr} 5 & 4 & 7 & 6 & 9 & 10 & 8 & 3 & 5 \\ -8 & -8 & -8 & -8 & -8 & -8 & -8 & -8 & -8 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} & & & & & \text{5.} \\ \begin{array}{rrrrrrrrrr} 6 & 5 & 9 & 3 & 8 & 7 & 8 & 2 & 4 \\ -7 & -7 & -7 & -7 & -7 & -7 & -7 & -7 & -7 \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} & & & & & \text{6.} \\ \begin{array}{rrrrrrrrrr} 5 & 6 & 7 & 8 & 10 & 12 & 11 & 4 & 3 \\ -10 & -10 & -10 & -10 & -10 & -10 & -10 & -10 & -10 \end{array} \\ \hline \end{array}$$

7.

12	11	10	9	8	7	6	5	4
3	3	3	3	3	3	3	3	3
<hr/>								

8.

11	10	12	8	9	7	5	4	6
9	9	9	9	9	9	9	9	9
<hr/>								

9.

6	2	1	12	3	10	11	0	12
11	11	11	11	11	11	11	11	11
<hr/>								

10.

0	12	8	10	9	11	5	7	3
5	5	5	5	5	5	5	5	5
<hr/>								

11.

6	5	4	3	2	1	0	12	13
12	12	12	13	12	12	12	12	12
<hr/>								

12.

1	2	3	4	5	6	7	8	9
12	11	10	9	8	7	6	5	4
<hr/>								

13.

12	11	10	9	8	7	6	5	4
1	2	3	4	5	6	7	8	9
<hr/>								

14.

6	7	8	9	10	11	12	1	2
1	2	3	4	5	6	7	8	9
<hr/>								

LESSON XXVI.

1. If a man earns \$4 per day, how much does he earn in 7 days?

Solution.—The equation is $7 \times \$4 = \28 . Therefore, in 7 days he earns 7 times \$4, or \$28.

2. How much will 6 barrels of fish cost, at the rate of \$8 a barrel?
3. How far can a man walk in 8 hours, if he walks 4 miles an hour?
4. How much must I pay for 9 quires of paper at 9 cents a quire?
5. If a man works 10 hours per day, how many does he work in 12 days?
6. At 20 cents a yard, how much will 8 yards of ribbon cost?
7. At the rate of 30 miles an hour, how far will a train go in 7 hours?
8. If a piece of work require 11 men 40 hours, how long will it require one man?
9. How much will 10 brooms cost at \$0.25 apiece?
10. There are 24 sheets in a quire. How many sheets in 10 quires?
11. If 1 acre produces 40 bushels of corn, how many bushels will 11 acres produce?
12. Twelve inches make 1 foot. How many inches make 30 feet?
13. One day has 24 hours. How many hours in 20 days?

LESSON XXVII.

1. What will 16 pairs of skates cost at \$3 a pair?
2. If the diameter of the earth is 8000 miles, how many thousands is three times that distance?
3. What will 19 yards of broadcloth cost at \$4 per yard?
4. At \$100 per acre what will 25 acres of ground cost?
5. Forty rods make 1 furlong. How many rods in 11 furlongs?
6. There are 60 minutes in 1 hour. How many minutes in 12 hours?
7. A boy on his bicycle goes 10 miles an hour. How far can he ride, at the same rate, in 17 hours?
8. When beef costs \$0.12 a pound what will 40 pounds cost?
9. What is the value of 25 street-car fares at 5 cents each?
10. What will 9 horses cost at \$120 each?
11. How many yards in twenty pieces of cloth, each containing 37 yards?
12. Forty-five lines, of fifty letters each, contain how many letters?
13. If there are 12 ounces in a pound, how many ounces are there in 12 pounds? In 30 pounds?
14. New York is 90 miles from Philadelphia; what is six times the distance?
15. What is the cost of 300 pounds of sugar at \$.05 a pound?

LESSON XXVIII.

1. How much can a man earn in 40 days, working 8 hours a day, at 20 cents an hour?

Solution.—1. The process is $40 \times 8 \times 20 = 320 \times 20 = 6400$.

2. Working 8 hours a day, in 40 days he works 40 times 8 hours, or 320 hours.

3. Earning 20 cents an hour, in 320 hours he earns 320 times 20 cents, or 6400 cents = \$64.

2. A book has 200 pages, each page has 25 lines, and each line 12 letters. How many letters in the book?

3. There are 4 quarts in a gallon and 2 pints in a quart. How many pints in 5 gallons?

4. Find the cost of 30 dozen pairs of shoes at \$3 a pair?

5. A basket contains 90 apples; each apple is worth 3 cents. Find the cost of 6 like baskets.

6. Twenty boards, each containing 15 square feet, are worth 10 cents a square foot. Find their value?

7. Answer promptly the following inquiries:

$$4 \times 9 \times 5 = ? \quad 5 \times 12 \times 10 = ? \quad 2 \times 28 \times 10 = ?$$

$$10 \times 7 \times 11 = ? \quad 30 \times 20 \times 10 = ? \quad 9 \times 40 \times 50 = ?$$

$$5 \times 6 \times 7 = ? \quad 40 \times 50 \times 20 = ? \quad 20 \times 12 \times 2 = ?$$

$$6 \times 12 \times 7 = ? \quad 5 \times 60 \times 20 = ? \quad 60 \times 12 \times 6 \times 10 = ?$$

$$8 \times 9 \times 10 = ? \quad 12 \times 10 \times 8 = ? \quad 700 \times 25 \times 20 = ?$$

8. Find the cost of 6 bags of coffee, each containing 65 pounds, at 20 cents a pound.

9. Mr. Jones has 17 bags of clover-seed, each containing 2 bushels. How much is his clover-seed worth at \$5 a bushel?

LESSON XXIX.

1. A farmer sold 6 cows at \$30 each and 4 horses at \$125 each. How much did he receive for all?

Solution.—1. The process is $6 \times 30 + 4 \times 125 = 180 + 500 = 680$.

2. At \$30 each he received for the cows 6 times \$30, or \$180.

3. At \$125 each he received for the horses 4 times \$125, or \$500.

4. He, therefore, received for all \$180 + \$500, or \$680.

2. What is the cost of 3 oranges at 4 cents each and 5 bananas at 2 cents each?

3. Find how much a boy paid for 3 slates at 12 cents apiece and 4 pencils at 5 cents apiece.

4. A teacher bought 12 blank books at 12 cents each and 10 slates at 10 cents each. Find the total cost.

5. I rode in my carriage 6 hours at the rate of 8 miles an hour, and went on foot 7 hours at the rate of 4 miles an hour. How far did I travel? How much farther did I ride than walk?

6. Find the value of the following:

- | | |
|--|--------------------------------------|
| 1. $4 \times 6 + 7 \times 9.$ | 6. $7 \times 24 - 18 \times 8.$ |
| 2. $4 \times 5 \times 6 + 8 \times 9.$ | 7. $8 \times 18 + 8 \times 12.$ |
| 3. $6 \times 5 \times 8 - 10 \times 8 \times 2.$ | 8. $64 \times 4 + 38 \times 4.$ |
| 4. $200 - 8 \times 15 + 12 \times 6.$ | 9. $4 \times 65 - 18 \times 12.$ |
| 5. $32 \times 8 + 105 \times 5.$ | 10. $10 \times 8 + 8 \times 60 + 6.$ |

7. Find the cost of 9 yards of ribbon at 9 cents a yard and 7 yards of ribbon at 7 cents a yard?

8. Find the cost of 6 sheep at \$6 a head, 80 thousand bricks at \$7 a thousand, and 9 cords of wood at \$3 a cord,

LESSON XXX.

The Parenthesis or Vinculum.

The Parenthesis, (), or the Vinculum, ——, is a sign of aggregation, and denotes that the expression enclosed by it is the first to be simplified. In $(5 - 3 + 8) \times 6 - 3$, or $\underline{5 - 3 + 8} \times 6 - 3$, we first find the value of $5 - 3 + 8$, which is 10. The result, 10, we then multiply by 6, and obtain 60. We finally say $60 - 3 = 57$.

1. Find the sum of 23 times 56 and 17 times 56.

Solution.—1. The process is $(23 + 17) \times 56 = 40 \times 56 = 2240$.

2. 23 times 56 and 17 times 56 are 40 times 56.
3. 40 times 56 = 2240.

2. If a man earns \$16 a week, and pays \$5 a week for his board, how much can he save in 9 weeks?

Solution.—1. The process is $(16 - 5) \times 9 = 11 \times 9 = 99$.

2. Earning \$16 a week and paying \$5 he saves \$16 - \$5 = \$11.

3. Therefore, in 9 weeks he can save 9 times \$11, or \$99.

3. If I buy eggs at 20 cents a dozen, and sell them at 25 cents a dozen, how much do I gain on 11 dozen?

4. A bicycler, on his wheel, averages 12 miles an hour. If on Monday he rides 9 hours, on Tuesday 10 hours, and on Wednesday 11 hours, how many miles does he ride in all?

5. Jefferson earns \$15 per week and pays \$6 for board and \$4 per week for other expenses. How much does he save in 52 weeks or one year?

6. Two trolley cars run in opposite directions from the same point, at the rate of 11 miles and 12 miles per hour, respectively. How far apart will they be in 6 hours?

7. Two employes receive \$16 per week each, but expend weekly \$11 and \$8, respectively. How much do they save in 13 weeks?

8. An accountant earns \$600 a year and expends \$475 a year. How much does he save in 6 years?

9. One basket contains 53 apples, another 63, and a third 7 times the difference of these numbers. How many apples in the 3 baskets?

10. Two pedestrians are 90 miles apart and approach each other, one at the rate of 6 miles per hour and the other at the rate of 5 miles per hour. How far apart will they be at the end of 7 hours?

LESSON XXXI.

1. If 1 yard of crape costs \$.40, what will 9 yards cost?

2. A milkman buys milk at 12 cents a gallon (4 quarts) and retails it at 5 cents a quart. Find his gain on a 10-gallon can.

3. A farmer sold 50 barrels of apples at \$3 per barrel, 70 barrels of potatoes at \$2 per barrel, and received in payment 30 barrels of flour at \$6 a barrel, and the balance in cash. How much cash did he receive?

4. If I bought meat at 20 cents a pound, at one time 13 pounds and at another time 19 pounds, what was the total cost?

5. A courier drives for 3 days at the rate of 9 miles per hour; the first day he is on the road 6 hours, the

second day 8 hours, and the third day 10 hours. Find the total distance he travels.

6. If a man buys 13 tons of hay at \$13 a ton, and sells 9 tons at \$19 a ton, and the rest at \$9 a ton, what does he gain by the transaction?

7. If coal costs \$7 a ton, and three tons last a month, how much will coal sufficient for 7 months cost?

8. A farmer bought 16 sheep; he sold 9 of them at \$5 apiece and the rest at \$4 apiece, and thus gained \$9. How much did he pay for the sheep?

9. How many are 7 times 9, plus 13, minus 7?

10. How many are 16 times 9 times 15, and 8 times 15?

11. Read and find the value of:

- | | |
|-------------------------------|--|
| 1. $3 \times (8 - 6)$. | 6. $(6 - 2) \times (7 - 4)$. |
| 2. $3 \times 8 - 6$. | 7. $\underline{10 + (2 \times 7)} - 12 - 6$. |
| 3. $4 \times (10 + 4)$. | 8. $\underline{\underline{5 + 6}} \times 7 - (10 - 3)$. |
| 4. $(5 + 4) \times (3 + 5)$. | 9. $\underline{\underline{14 + 16}} \times 4 - 29$. |
| 5. $5 + (4 \times 3) + 5$. | 10. $(21 - 10) \times 6 - 2 \times 33$. |

12. For rapid work.

1. $(26 - 4 + 8) \times 4 + (9 \times 6) - 28 + 16 - 22 + 10$.
2. $9 \times (7 - 4) + (6 \times 3) - (5 \times 3) + 45 - 15 - 38$.
3. $(10 + 8 - 3) \times 5 + 8 + 7 - 45 + (5 \times 30)$.
4. $9 + 6 - 3 + 15 + (3 \times 6) + 15 - 6 + 27 + 8$.
5. $(15 \times 7) + (10 \times 8) - 60 - 20 + 56 - 10 + (19 \times 3)$.

NOTE.—Pupils will observe that in Arithmetic, as in Algebra, the terms of an arithmetical expression are the parts connected by the signs *plus* and *minus*. It is for this reason that in the foregoing exercises the numbers to be multiplied are arranged as you see them. No. 4, in 11, is read “The product of the parentheses 5 plus 4 and 3 plus 5.” No. 3, in 12, is read “5 times the parentheses 10 plus 8 and minus 3, plus 8, plus 7, minus 45, plus 5 times 30.”

LESSON XXXII.

Division.

Division is the process of finding how many times one number contains another, or of dividing a number into equal parts.

The **Sign of Division** is \div . Placed between two numbers, it implies that the number preceding it is to be divided by the number following it. $8 \div 4 = 2$, is read “8 divided by 4 equals 2.”

In the equation $8 \div 4 = 2$, 8 is the **Dividend**, 4 is the **Divisor**, and 2 is the **Quotient**.

Division may be considered as a short process of Subtraction, for it is plain that the number of times 2 is contained in 8 is the number of times 2 can be subtracted from 8.

Division is the reverse of Multiplication. Because 7 times 8 are 56, we know that 56 contains 8 seven times and 7 eight times. Recite and compare the following:

Multiplication.

- $5 \times 2 = 10$; therefore, $10 \div 2 = 5$ and $10 \div 5 = 2$.
- $5 \times 3 = 15$; therefore, $15 \div 3 = 5$ and $15 \div 5 = 3$.
- $5 \times 4 = 20$; therefore, $20 \div 4 = 5$ and $20 \div 5 = 4$.
- $5 \times 5 = 25$; therefore, $25 \div 5 = 5$.
- $5 \times 6 = 30$; therefore, $30 \div 6 = 5$ and $30 \div 5 = 6$.
- $5 \times 7 = 35$; therefore, $35 \div 7 = 5$ and $35 \div 5 = 7$.
- $5 \times 8 = 40$; therefore, $40 \div 8 = 5$ and $40 \div 5 = 8$.
- $5 \times 9 = 45$; therefore, $45 \div 9 = 5$ and $45 \div 5 = 9$.
- $5 \times 10 = 50$; therefore, $50 \div 10 = 5$ and $50 \div 5 = 10$.
- $5 \times 11 = 55$; therefore, $55 \div 11 = 5$ and $55 \div 5 = 11$.

Multiplication.Division.

$5 \times 12 = 60$; therefore, $60 \div 12 = 5$ and $60 \div 5 = 12$.

$5 \times 13 = 65$; therefore, $65 \div 13 = 5$ and $65 \div 5 = 13$.

SUGGESTION.—Pupils should be required to write neatly upon the blackboard exercises similar to the above; and, after thorough drill, these tables should be erased and purely mental exercises given in the following manner: The teacher announces, "9 times 8," and immediately a pupil takes these numbers, names their product, and using the factors 9 and 8 as divisors, gives the quotients. The pupil's answer should be, "9 times 8 equals 72; therefore, 72 divided by 8 equals 9, and 72 divided by 9 equals 8." This is a valuable exercise, and should be carried beyond 12 times 12.

LESSON XXXIII.

The two-fold purpose of Division, as indicated in the definition, gives rise to two forms of solution. Both should be carefully studied.

1. At 3 cents each, how many oranges can be bought for 21 cents?

Solution.—1. The process is $21 \text{ cts.} \div 3 \text{ cts.} = 7$. 2. At 3 cents apiece 21 cents will buy as many oranges as 3 cents is contained times in 21 cents. 3 cents is contained in 21 cents 7 times. 3. Therefore, 21 cents will buy 7 oranges at 3 cents apiece.

NOTE.—Observe that in this problem the dividend and the divisor are like numbers, both being cents, and the solution simply shows how many times one number contains another number like it. The quotient is an abstract number, and is subsequently applied to oranges. This form of solution is applicable to all similar problems.

2. Distribute \$35 equally among 7 persons.

Solution.—1. The process is $\$35 \div 7 = \5 . 2. If $\$35$ be divided equally among 7 persons, one person will receive one of the seven equal parts, or one-seventh of $\$35$, which is $\$5$. 3. Therefore, each person will receive $\$5$.

NOTE.—Here you will observe the dividend and the quotient are like numbers and the divisor is an abstract number, simply showing the number of equal parts into which the dividend is to be divided.

Apply these forms of solution to the following problems:

3. At 6 cents apiece, how many melons can be bought for 42 cents?
4. Lucy spent 64 cents for oranges. If they cost 8 cents apiece, how many did she buy?
5. If John walks at the rate of 7 miles a day, how long will he be in walking 56 miles?
6. How many hats at \$4 each can be bought for \$36?
7. Harry divided 35 marbles among his playmates, giving to each 7 marbles. How many playmates were there?
8. What is the price of a barrel of flour when 9 barrels cost \$54?
9. If \$36 will buy 9 yards of cloth, what is the price per yard?
10. Laura rode her bicycle 40 miles in 4 days. What was the average per day?
11. A house rents for \$108 a year. What is the monthly rental?
12. When 96 cents will buy 8 dozen eggs, what is the price per dozen?

LESSON XXXIV.

1. A field of 80 acres was divided into lots containing 8 acres each. How many lots were thus made?
2. If a train ran 9 miles an hour, in how many hours would it run 72 miles?
3. Fifty-four seats were arranged in 6 equal rows. How many seats were in each row?
4. Sixty-four dollars will pay for how many rods of fence at \$8 per rod?
5. In 9 hours a bicyclist rode 81 miles? What was his rate of speed per hour?
6. At the rate of 9 cents to each, how many beggars would receive 63 cents?
7. How many melons, at 12 cents apiece, may be bought for 96 cents?
8. For \$88, and at the rate of \$8 for each, a farmer sold how many sheep?
9. A dealer gave coal worth \$6 a ton for wheat worth \$72. Find the number of tons he gave.
10. If 12 pounds of sugar cost 132 cents, how many cents does one pound cost?
11. If a dozen oranges are worth 48 cents, what is the value of one orange?
12. In 63 days there are how many weeks?
13. There are 12 inches in one foot. How many feet in 144 inches?
14. What is the cost of one ton of coal, if 11 tons cost \$121?
15. How long will it take a man to earn \$132, at \$11 a week?

16. A flight of stairs mounts 128 inches. Find the number of steps in the flight if each step is 8 inches high.

17. In 7 weeks there are 49 days. How many days are there in one week?

18. How much do 5 doors cost if I pay \$56 for 8 of them?

19. When coal is worth \$6.60 a ton, how many tons can be bought for \$660.

20. When flour is \$6 a barrel, how many barrels of flour will \$1200 buy?

LESSON XXXV.

1. If 5 books cost \$1.50, what do they cost apiece?

Solution.—1. The process is $\$1.50 \div 5 = \0.30 .

2. Since 5 books cost \$1.50, one book costs one-fifth of \$1.50, or \$0.30.

2. When 40 pounds of butter cost \$8, what does 1 pound cost?

Solution.—1. The equations are $\$8 = 800$ cents; $800 \div 40 = 20$.

2. Since 40 pounds cost 800 cents, 1 pound costs one-fortieth of 800 cents, or 20 cents.

3. Sixty oranges cost \$2.40. Find the cost of each.

4. I paid \$5 for 20 dozen eggs. What did they cost me per dozen?

5. How long will it take a man to earn \$120 at \$20 a week?

6. If 30 horses cost \$6000, what is the average cost per head?

7. If a box will hold 200 oranges, how many boxes will hold 8000 oranges?
 8. There are 60 minutes in 1 hour. How many hours in 3600 minutes?
 9. At 9 cents a pound, how many pounds of rice can be bought for \$18?
 10. At 40 cents a pound, how many pounds of coffee can be bought for \$9.60?
 11. If 6 writing desks cost \$54, what cost 9 writing desks at the same rate?
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LESSON XXXVI.

1. When oranges cost 4 cents each, a boy gave 80 cents for 8 oranges and 12 apples. How much did he pay apiece for the apples?

Solution.—1. The process is $(80 - 8 \times 4) \div 12 = 4$.

2. At 4 cents each, 8 oranges cost 32 cents.

3. 80 cents — 32 cents = 48 cents remaining.

4. Since 12 apples cost 48 cents, 1 apple costs $\frac{1}{12}$ of 48 cents, or 4 cents.

2. A woman exchanged berries for sugar, giving 10 quarts of berries, at 8 cents a quart, and 25 cents in money for 21 pounds of sugar. What did the sugar cost per pound?

3. If a man receives 50 pounds of sugar in exchange for 25 pounds of cheese at 10 cents a pound, what is the price of the sugar per pound?

4. If 6 arithmetics, at 40 cents each, and 5 histories together cost \$5, how much do the histories cost apiece?

5. Mr. A., having earned \$70, paid a debt of \$25. How many yards of cloth at \$3 a yard can he buy with the remainder of his money?

6. A tank holding 600 gallons of water is full. While one pipe brings into it 300 gallons per hour, another discharges 400 gallons. At the end of 4 hours how many gallons will remain in the tank?

7. If 14 sheep cost \$98, for how much must 5 of them be sold to gain \$5 on the sale?

8. When 4 bushels of wheat make 1 barrel of flour, which sells for \$8, what is a bushel of wheat worth?

LESSON XXXVII.

1. If 12 men can do a piece of work in 6 days, how long will it take 9 men to do it?

Solution.—1. The process is $(12 \times 6) \div 9 = 8$.

2. Since 12 men do the work in 6 days, it will take 1 man 12 times 6 days, or 72 days.

3. Hence, 9 men can do the work in $\frac{1}{3}$ of 72 days, or 8 days.

2. A piece of work can be done by 21 men in 5 days; how long will it take 15 men to do the same work?

3. A quantity of bread will keep a family of 10 persons three weeks. How many days will it last when 4 persons are absent?

4. If 15 men can build a wall in 6 days, how many men will be required to build the wall in 9 days? How many days must 18 men work to build the wall?

5. A certain quantity of corn lasts 11 horses 21 days. How many horses will it supply for 7 days?
 6. If 24 men can build a house in 16 days, how long will it take 12 men?
 7. If 9 boys can do a piece of work in 12 days, how many boys will do the work in 18 days?
 8. A wall can be built by 21 men in 15 days, how long will it take 45 men to build the wall?
 9. If 6 men can build a house in 12 days of 10 hours each, how many men must be added if the work is to be done in 8 days of 9 hours each?
 10. If 30 sheep consume the pasture of a 10-acre field in 12 weeks, how many sheep will 10 acres feed for 15 weeks?
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LESSON XXXVIII.

1. What is the cost of 14 hats and 13 caps, if hats are worth \$4 apiece and caps one-half as much?
2. Suppose a trader, who has 12 barrels of flour on hand, should lay out \$35 in buying more flour at \$5 a barrel. How many barrels would he then have?
3. How many pounds of sugar, at 20 cents a pound, must be given for 5 pieces of cotton-cloth, containing 40 yards each, at 40 cents a yard?
4. Four school-rooms contain, on an average, 75 pupils per room. If 128 pupils are boys, how many are girls, and what is the average number of girls in each room?
5. A boy earns half as much as a man. A man earns \$2 a day. What will 9 men and 6 boys earn in 11 days?

6. Mary can earn as much in 2 days as her sister can in 3 days. How much should Mary receive when her sister receives 72 cents per day?

7. A laborer bought a pair of shoes costing him \$2.50 and 6 yards of cloth at 65 cents per yard, and gave therefore for his labor for 4 days and 60 cents in cash. What was the value of his wages per day?

8. Each fore-wheel of a carriage is 8 feet in circumference and each hind-wheel is 12 feet. How many more revolutions will the fore-wheels make than the hind-wheels in going 480 feet?

9. Mr. Jackson sold some cords of wood for \$450, and thus gained \$3 per cord. The wood sold cost him \$300. Find the number of cords and the cost per cord?

10. Six pounds of coffee worth 40 cents a pound is mixed with 4 pounds of coffee worth 30 cents a pound. The mixture was sold so as to gain 4 cents on each pound? For how much was the mixture sold?

LESSON XXXIX.

1. Two boys had 180 marbles, and one of the boys had twice as many as the other. How many had each?

Solution.—1. The equation is: Once the smaller number + twice the smaller number = 3 times the smaller number.

2. Hence, 3 times the smaller number = 180.
3. Once the smaller number = 60.
4. Twice the smaller number = 120.
5. Therefore, one boy had 60 and the other had 120 marbles.

2. Two hogs weigh together 570 pounds, and one weighs twice as much as the other. Find the weight of each.
3. A. paid \$8 for grapes and apples, giving 3 times as much for apples as for grapes; how much for each?
4. I have 36 dimes and nickels, but three times as many nickels as dimes. How many of each have I?
5. Of 300 pupils there are twice as many girls as boys. How many of each?
6. Two droves contain 310 cattle; one drove has 4 times as many as the other. How many in each?
7. A. and B. owe C. \$169, but A. owes him 12 times as much as B. Find the debt of each?
8. A lady bought 54 yds. of silk and lace; 5 times as much lace as silk. How many yards of silk?
9. A. takes one step while B. takes 2. When both have taken 198 steps, how many has each taken?

LESSON XL.

1. John, William, and Harry together have 192 marbles. How many has each, if John has twice and William three times as many as Harry?

Solution.—By the conditions of the problem all have 6 times the number Harry has. Hence Harry has one-sixth of 192 marbles, or 32 marbles; and since John has 2 times and William 3 times as many as Harry, they have 64 marbles and 96 marbles respectively.

NOTE.—By blackboard illustration show pupils the forms of solutions in this lesson. These will be helpful in purely mental work.

2. A man bought 156 fowls. If there were 3 times as many ducks as turkeys, and twice as many chickens as both ducks and turkeys, how many of each did he buy?

3. A., B., and C, do a piece of work for \$360. If B. worked twice as many days as A., and C. two and one-half times as many days as B., how much money did each receive?

4. Divide \$315 among A., B., C., and D., giving to A. a certain sum and to each of the others in succession twice as much as to the one immediately preceding.

5. A gentleman bought three horses for \$1100, paying for the first \$50 more than for the second, and for the second \$30 more than for the third. How much did he pay for each horse?

SUGGESTION.—1 time cost of 3d = cost of 3d.

1 time cost of 3d + \$30 = cost of 2d.

1 time cost of 3d + \$80 = cost of 1st.

Hence 3 times cost of 3d + \$110 = cost of all, or \$1100.

6. Three men built a wall 120 rods long. How many rods did each man build, if A. built 12 rods less B., and C. built 6 rods less than A.?

7. If John has \$20 less than twice as much as James has, and James has \$50 more than twice as much as Charles, how much has each, if all have \$515?

8. A man paid \$195 for work done by 7 men, 4 women, and 10 boys. How much did each person receive, if each man earned one and one-half times as much as each woman, and each woman twice as much as each boy?

To THE TEACHER.—It might be well to review before taking up Common Fractions.

COMMON FRACTIONS.

LESSON I.

A Fraction is a part of anything, and is expressed by placing one number above another with a line between them.

The number above the line is the Numerator and the number below the line is the Denominator.

The *Denominator* names the fraction.

Halves.

1. If anything be divided into two equal parts, one of the parts is called one-half.
2. The fraction one-half ($\frac{1}{2}$) denotes one of the two equal parts of any unit.
3. How many halves are there in a unit? In 2 units?
4. How many halves in 3 units? In 5? In 7? In 9?
5. If a melon be cut into two equal parts, each part will be what fraction of the melon?
6. If one-half a cord of wood cost \$3, how much will 1 cord cost? 6 cords? 8 cords? 10 cords?
7. How many ones or units in these: $\frac{4}{2}$, $\frac{6}{2}$, $\frac{8}{2}$, $\frac{10}{2}$, $\frac{16}{2}$, $\frac{20}{2}$, $\frac{18}{2}$, $\frac{46}{2}$, $\frac{50}{2}$, $\frac{36}{2}$?
8. What is one-half of each of the following numbers: 2, 8, 4, 12, 10, 16, 14, 20, 18, 22?
9. Each of the following numbers is one-half of some number; state in each case what that number is: 11, 9, 10, 7, 8, 5, 6, 2, 4, 1.

10. State the number of halves in each of the following numbers: 13, 17, 19, 23, 29, 31, 37, 47, 70, 100.

11. How many halves in 5 and 1 half?

Solution.— $1 = \frac{2}{2}$; $5 = \frac{10}{2}$; $\frac{10}{2} + \frac{1}{2} = \frac{11}{2}$. Therefore, 5 and $\frac{1}{2} = \frac{11}{2}$.

12. How many halves in each of these: $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $6\frac{1}{2}$, $7\frac{1}{2}$, $8\frac{1}{2}$, $9\frac{1}{2}$, $11\frac{1}{2}$?

13. How many units or ones are there in 13 halves?

Solution.—1. The process is $\frac{13}{2} = 6\frac{1}{2}$.

2. Since 2 halves = 1, 13 halves = as many ones as 2 is contained times in 13, which is $6\frac{1}{2}$ times.

3. Therefore, $\frac{13}{2} = 6\frac{1}{2}$.

14. How many ones are there in each of these: $\frac{11}{2}$, $\frac{9}{2}$, $\frac{15}{2}$, $\frac{17}{2}$, $\frac{19}{2}$, $\frac{21}{2}$, $\frac{23}{2}$, $\frac{25}{2}$, $\frac{27}{2}$, $\frac{29}{2}$?

15. Name instantly one-half of each of the following: 7, 5, 3, 13, 31, 35, 37, 51, 57, 75.

LESSON II.

Thirds.

1. The fraction one-third ($\frac{1}{3}$) denotes one of the three equal parts of a unit.

2. How many thirds are there in 1? In 3? In 5? In 7?

3. How many thirds in each of these: 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 30, 33, 44, 50, 60?

4. If $\frac{1}{3}$ of a yard of ribbon is worth 7 cents, what is a whole yard worth? How much are 3 yards worth?

5. If $\frac{1}{3}$ of a pound of confections costs 20 cents, what will 2 pounds cost?
6. If $\frac{1}{3}$ of a day's labor is worth 30 cents, what will 5 days' labor cost?
7. Name the number of units in each of these: $\frac{6}{3}$, $\frac{12}{3}$, $\frac{15}{3}$, $\frac{21}{3}$, $\frac{18}{3}$, $\frac{27}{3}$, $\frac{33}{3}$, $\frac{48}{3}$, $\frac{57}{3}$, $\frac{63}{3}$, $\frac{24}{3}$, $\frac{36}{3}$, $\frac{45}{3}$, $\frac{42}{3}$, $\frac{54}{3}$, $\frac{51}{3}$, $\frac{99}{3}$, $\frac{96}{3}$, $\frac{48}{3}$, $\frac{132}{3}$.
8. How many thirds in $2\frac{2}{3}$? In $1\frac{1}{3}$? In $3\frac{2}{3}$?
9. Find the number of thirds in the following: $1\frac{1}{3}$, $2\frac{2}{3}$, $3\frac{2}{3}$, $5\frac{1}{3}$, $7\frac{2}{3}$, $8\frac{1}{3}$, $10\frac{2}{3}$, $15\frac{1}{3}$, $4\frac{1}{3}$, $5\frac{2}{3}$, $7\frac{1}{3}$, $6\frac{2}{3}$, $10\frac{2}{3}$, $0\frac{1}{3}$, $11\frac{1}{3}$, $30\frac{2}{3}$.
10. To how many units is each of these equal: $\frac{5}{3}$, $\frac{7}{3}$, $\frac{11}{3}$, $\frac{13}{3}$, $\frac{16}{3}$, $\frac{17}{3}$, $\frac{19}{3}$, $\frac{20}{3}$, $\frac{23}{3}$, $\frac{25}{3}$, $\frac{20}{3}$, $\frac{28}{3}$, $\frac{44}{3}$, $\frac{52}{3}$, $\frac{64}{3}$, $\frac{68}{3}$, $\frac{76}{3}$, $\frac{80}{3}$, $\frac{92}{3}$, $\frac{100}{3}$.
11. If $\frac{2}{3}$ of a ton of coal is worth \$4, how much is a ton worth?
12. If an acre of land cost \$75, what is the cost of $\frac{2}{3}$ of an acre?
13. If a man pays \$66 for a watch and $\frac{2}{3}$ as much for a clock, how much does he pay for the clock?
14. If there are 8 inches in $\frac{2}{3}$ of a foot, how many inches are there in a whole foot?
15. If a bicycler rides 14 miles in $2\frac{1}{2}$ hours, how far does he ride in one hour?
- Solution.**— $2\frac{1}{2}$ hours = $\frac{7}{2}$ hours; in $\frac{7}{2}$ hours he rides 14 miles; in $\frac{1}{2}$ of an hour he rides $\frac{1}{7}$ of 14 miles or 2 miles; in $\frac{2}{3}$ of an hour, 3 times 2 miles, or 6 miles. Therefore, in 1 hour he rides 6 miles.
16. If $2\frac{2}{3}$ yards of cloth cost \$8, how much does 1 yard cost?
17. If $6\frac{1}{3}$ pounds of sugar cost 57 cents, how much does 1 pound cost?

18. If a man can earn 40 cents in $2\frac{1}{2}$ hours, how much can he earn in 10 hours?
19. A man earns \$100 in $3\frac{1}{2}$ months; how much can he earn in 12 months?
20. A boy can walk 16 miles in $5\frac{1}{2}$ hours; how far can he walk in 12 hours?
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LESSON III.

Fourths.

1. How many fourths are in each of the following numbers: 1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22?
2. How many single things are represented by $\frac{4}{4}$, $\frac{8}{4}$, $\frac{12}{4}$, $\frac{24}{4}$, $\frac{20}{4}$, $\frac{16}{4}$, $\frac{36}{4}$, $\frac{32}{4}$, $\frac{44}{4}$, $\frac{56}{4}$?
3. State the number of fourths in each of these: $1\frac{1}{4}$, $4\frac{2}{4}$, $5\frac{3}{4}$, $6\frac{1}{4}$, $2\frac{3}{4}$, $7\frac{1}{4}$, $8\frac{2}{4}$, $11\frac{3}{4}$.
4. Change the following to fourths: $3\frac{3}{4}$, $2\frac{2}{4}$, $1\frac{3}{4}$, $5\frac{2}{4}$, $12\frac{1}{4}$, $6\frac{1}{4}$, $9\frac{3}{4}$, $11\frac{3}{4}$.
5. How many units in these: $\frac{13}{4}$, $\frac{22}{4}$, $\frac{17}{4}$, $\frac{15}{4}$, $\frac{23}{4}$, $\frac{19}{4}$, $\frac{27}{4}$, $\frac{39}{4}$, $\frac{31}{4}$, $\frac{33}{4}$, $\frac{49}{4}$, $\frac{57}{4}$, $\frac{69}{4}$, $\frac{71}{4}$, $\frac{82}{4}$, $\frac{90}{4}$.
6. If $\frac{1}{4}$ of a pound of butter is worth 7 cents, how much is a pound worth?
7. If $\frac{4}{4}$ of a yard equal 27 inches, how many inches are there in a whole yard?
8. If a carriage cost \$300 and a horse $\frac{1}{4}$ as much, what is the cost of the horse?
9. I bought $2\frac{1}{2}$ pounds of beef for 45 cents. What rate did I pay per pound?

Solution.— $2\frac{1}{4} = \frac{9}{4}$. Since $\frac{9}{4}$ of a pound cost 45 cents, $\frac{1}{4}$ of a pound cost $\frac{1}{9}$ of 45 cents, or 5 cents; since $\frac{1}{4}$ of a pound cost 5 cents, $\frac{1}{4}$ of a pound cost 4 times 5 cents, or 20 cents. Therefore, 1 pound cost 20 cents.

10. A man paid \$22 for $2\frac{3}{4}$ cords of wood. How much did he pay per cord?

11. Six and $\frac{1}{4}$ tons of coal cost me \$25. How much did I pay for 1 ton?

12. What is $\frac{1}{4}$ of these: 4, 8, 16, 20, 28, 30, 36, 40, 48?

13. What is $\frac{3}{4}$ of these: 12, 28, 32, 44, 52, 4, 24, 64, 100?

14. What is $\frac{1}{4}$ of these: 11, 15, 17, 19, 21, 25, 27, 29, 39?

15. How do you find $\frac{3}{4}$ of 9?

Solution.— $\frac{3}{4} = \frac{1}{4}$ of 3; $\frac{1}{4}$ of 3 times 9 = $\frac{1}{4}$ of 27 = $6\frac{3}{4}$.
Therefore, $\frac{3}{4}$ of 9 = $6\frac{3}{4}$.
Or, $\frac{1}{4}$ of 9 = $\frac{9}{4}$; $\frac{3}{4}$ of 9 = 3 times $\frac{9}{4} = \frac{27}{4} = 6\frac{3}{4}$.

16. In like manner find $\frac{3}{4}$ of the following numbers:
13, 11, 15, 17, 7, 5, 19, 21, 23, 25.

17. A lady, in shopping, spent $\frac{3}{4}$ of \$20. How much did she spend?

18. If $1\frac{3}{4}$ bushels of wheat weigh 105 pounds, how much does $\frac{1}{4}$ of a bushel weigh? How much 1 bushel?

19. What must be paid for a bushel of oats if $3\frac{3}{4}$ of a bushel cost 75 cents?

20. A farmer, having 21 gallons of cider, sold $\frac{3}{4}$ of it. How many gallons did he sell?

21. A lady bought a bicycle for \$60 and sold it for $\frac{3}{4}$ of what it cost. What did she get for it?

22. At the rate of $5\frac{1}{4}$ pounds of mutton for \$1.05, what does 1 pound cost?

LESSON IV.

Fifths.

1. Tell promptly the number of fifths in each of the following numbers: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 18, 16, 14, 12, 10, 8, 6, 4, 2.

2. How many units do you perceive in $\frac{5}{5}$? $\frac{20}{5}$? $\frac{35}{5}$? $\frac{55}{5}$? $\frac{45}{5}$? $\frac{15}{5}$? $\frac{30}{5}$? $\frac{25}{5}$? $\frac{65}{5}$? $\frac{75}{5}$?

3. How many fifths are in $1\frac{1}{5}$? $3\frac{3}{5}$? $5\frac{4}{5}$? $7\frac{2}{5}$? $9\frac{3}{5}$? $11\frac{4}{5}$? $13\frac{2}{5}$? $15\frac{1}{5}$? $2\frac{4}{5}$? $4\frac{3}{5}$? $8\frac{1}{5}$? $6\frac{4}{5}$? $4\frac{1}{5}$? $10\frac{2}{5}$? $8\frac{4}{5}$? $12\frac{3}{5}$?

4. What unit and fraction of a unit in each of these: $\frac{2}{5}$, $\frac{13}{5}$, $\frac{19}{5}$, $\frac{16}{5}$, $\frac{18}{5}$, $\frac{21}{5}$, $\frac{32}{5}$, $\frac{36}{5}$?

5. A boy lost 9 marbles, which was $\frac{3}{5}$ of all he previously had. How many had he?

6. When $\frac{1}{5}$ of a ton of coal costs \$1.20, what is the cost of a ton?

7. If $\frac{3}{5}$ of the cost of a dictionary is \$6, what is its cost?

8. Find $\frac{3}{5}$ and $\frac{4}{5}$ of each of these: 10, 15, 20, 25, 100, 80.

9. How do you find $\frac{4}{5}$ of 21?

Solution.— $\frac{4}{5} = \frac{1}{5}$ of 4; $\frac{1}{5}$ of 4 times 21 = $\frac{1}{5}$ of $\frac{84}{5} = 16\frac{4}{5}$. Therefore, $\frac{4}{5}$ of 21 = $16\frac{4}{5}$.

Or, $\frac{1}{5}$ of 21 is $\frac{21}{5}$; $\frac{4}{5}$ of 21 = 4 times $\frac{21}{5} = \frac{84}{5} = 16\frac{4}{5}$.

10. Find $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ of: 11, 23, 19, 7, 14, 18, 30, 20, 4, 8, 6, 9, 13, 31, 48, 57.

11. A lady has lived 55 years; another has lived only $\frac{4}{5}$ as long. What is the age of the latter?

12. A farmer sold $\frac{2}{5}$ of 100 bushels of corn to one man, and $\frac{3}{5}$ of it to another. How many bushels did he sell to each man? How many bushels to both men?
13. How long do I work, if I work $\frac{4}{5}$ of 12 hours?
14. What is the cost of $\frac{3}{5}$ of a piece of cloth, if the whole piece is worth \$40.
15. A dozen eggs were sold for 24 cents, which was $\frac{6}{5}$ of their cost. What was the cost?
16. Five equal lots of ground contain 1 acre. Three of the lots were sold for \$240. The rate being the same, what would the remaining lots sell for, and the whole acre?
17. If $\frac{4}{5}$ of a week's work is worth \$20, what is the work of a whole week worth?
18. Of the words assigned him, a boy spelled correctly 60, which was $\frac{3}{5}$ of the whole number. Find the whole number.
19. Mr. Kingston is 45 years old, and his wife $\frac{4}{5}$ as old. How old is his wife?
20. A horse cost \$125, and $\frac{4}{5}$ of the cost of the horse is 5 times the cost of the harness. What did the harness cost?
21. A horse cost \$150 and a wagon $\frac{3}{5}$ as much. How much did the wagon cost?
22. If 45 is $\frac{3}{5}$ of a number, what is the number?
23. From New York to Philadelphia is 90 miles. When you have travelled $\frac{4}{5}$ of the distance, how many miles have you gone?
24. One mile equals 320 rods; how many rods are there in $\frac{3}{5}$ of a mile?
25. If the diameter of the earth is 8000 miles, how far from the centre is a point $\frac{2}{5}$ of 8000 miles from the surface?

LESSON V.

Miscellaneous.

1. What is $\frac{1}{2}$ of each of the numbers, namely, 1, 2, 3, 4, 5, 6, 7, 8, 9?
2. What is $\frac{2}{3}$ of 15? 9? 6? 30? 12? 21? 18?
27? 24? 33? 36?
3. Find $\frac{3}{4}$ of 24, 100, 20, 36, 72, 40, 60, 120, 160.
4. Find $\frac{5}{6}$ of 15, 10, 20, 45, 55, 65, 75, 85, 95, 100.
5. What is $\frac{5}{8}$ of 12? 36? 18? 24? 54? 90? 96?
84? 72? 78?
6. Find the value of:
 1. $\frac{6}{7}$ of 14, 49, 105, 56, 63, 70, 77, 84, 91, 98.
 2. $\frac{7}{8}$ of 8, 64, 144, 112, 40, 56, 72, 88, 96, 32.
 3. $\frac{9}{10}$ of 100, 90, 80, 70, 110, 200, 130, 140, 150,
160.
 4. $\frac{11}{12}$ of 48, 36, 60, 24, 12, 72, 108, 96, 132, 84.
 5. $\frac{5}{6}$ of 1, 8, 16, 80, 104, 144, 112, 24, 32.
 6. $\frac{7}{10}$ of 1, 10, 100, 120, 130, 140, 150, 160, 170.
 7. $\frac{8}{11}$ of 11, 132, 22, 121, 33, 110, 44, 99, 55, 77.
 8. $\frac{2}{7}$ of 1, 84, 91, 14, 98, 21, 105, 28, 112, 35.
 9. $\frac{8}{9}$ of 81, 90, 99, 108, 72, 1, 9, 63, 54, 45.
 10. $\frac{2}{3}$ of 7, 11, 13, 16, 17, 19, 20, 22, 23, 25.
 11. $\frac{3}{5}$ of 6, 12, 9, 7, 21, 41, 52, 66, 77, 84.
 12. $\frac{5}{6}$ of 8, 11, 20, 13, 91, 14, 16, 17, 19, 21.
 13. $\frac{7}{8}$ of 9, 12, 22, 43, 57, 64, 75, 100, 60, 80.
7. There are 60 minutes in an hour. How many minutes are there in $\frac{1}{2}$ of an hour?
8. If a field is worth \$120, what is $\frac{2}{3}$ of it worth?
9. What will $\frac{3}{4}$ of 42 pounds of coffee cost at 30 cents per pound?

10. A ton of coal costs $\frac{1}{4}$ as much as a ton of hay, whose value is \$14. Find the cost of 9 tons of coal.
11. A turkey cost me \$2.40 and a duck $\frac{1}{2}$ as much. Find what both cost.
12. If $\frac{1}{3}$ of \$96 is the cost of 6 cords of wood, what is the cost of 2 cords?
13. If 63 is the sum of two numbers, and one of the numbers is $\frac{1}{3}$ of that sum, what is the other number?
14. A barrel of flour contains 196 pounds. $\frac{2}{3}$ of it was sold at 5 cents per pound. Find the value of what was sold.
15. John Adams lived to be 90 years old; his son, John Quincy Adams, lived to be $\frac{2}{3}$ as old. Find how long the latter lived.
16. In a day there are 24 hours. If John worked $\frac{1}{2}$ of a day and James $\frac{1}{3}$ of a day, how many hours did both work?
17. A grocer sells 144 eggs to three customers; to the first $\frac{1}{3}$ of them and to the second $\frac{1}{4}$ of them. How many were sold to the third buyer?
18. A barrel of beef contains 200 pounds. If one person buys $\frac{1}{4}$ of it and another $\frac{1}{5}$ of it, how much remains?
19. A ship had a crew of 144 men and added thereto $\frac{5}{12}$ as many. What was then the size of the crew?
20. A bicyclist rode on Monday 44 miles and on Tuesday $\frac{7}{11}$ of that distance. How far did he ride in the two days?
21. A farmer, having taken 154 fowls to market, sold in the forenoon $\frac{3}{7}$ of them and in the afternoon $\frac{5}{11}$ of the remainder. How many fowls had he then?

- 22.** A gentleman bought $1\frac{1}{2}$ of what the farmer had then at \$1.50 apiece, and presented in payment a ten-dollar bill. How much money did the buyer receive back?
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LESSON VI.

Reduction to Higher Terms.

1. How many fourths are there in one apple?
2. What is one-half of 4 fourths of an apple?
3. How many fourths are there in one-half?

Solution.—1. The equations are $1 = \frac{4}{4}$, and $\frac{1}{2} = \frac{2}{4}$.
 2. Since in 1 there are 4 fourths, in $\frac{1}{2}$ there are $\frac{1}{2}$ of $\frac{4}{4}$, or $\frac{2}{4}$.
 3. Therefore, there are $\frac{2}{4}$ in $\frac{1}{2}$, and $\frac{1}{2} = \frac{2}{4}$.

4. In like manner find how many *sixths* there are in $\frac{3}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{2}$, $\frac{5}{3}$, and fourths in $\frac{3}{2}$, $\frac{5}{2}$, $\frac{6}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$.

SUGGESTION.— $\frac{1}{2} = \frac{2}{4}$; $\frac{3}{2} = ?$

5. How many eighths are there in: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{3}{2}$, $2\frac{1}{2}$, $2\frac{1}{4}$, $3\frac{1}{4}$, $4\frac{1}{4}$, $5\frac{1}{2}$, $5\frac{1}{4}$?
6. Change to tenths the following: $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{2}$, $\frac{3}{5}$, $\frac{5}{2}$, $2\frac{1}{5}$, $3\frac{1}{2}$, $4\frac{2}{5}$, $5\frac{3}{5}$, $3\frac{4}{5}$.
7. Reduce to twelfths: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{6}$, $3\frac{1}{2}$, $5\frac{2}{3}$.
8. Reduce to sixteenths: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{7}{8}$, $\frac{8}{8}$, $2\frac{1}{2}$, $3\frac{3}{4}$, $4\frac{3}{8}$.
9. Since $\frac{1}{2} = \frac{2}{4}$, $\frac{1}{3} = \frac{2}{6}$, $\frac{2}{3} = \frac{4}{6}$, etc., does the change thus made in the form of the fraction change its value?
10. Does multiplying both terms of a fraction by the same number change its value?

11. By thus multiplying change $\frac{1}{2}$ to eighteenths.

1. The equations are $1 = \frac{18}{18}$; $\frac{1}{2} = \frac{9}{18}$.

2. One-half of $\frac{1}{8}$ is $\frac{9}{18}$, and taking both terms of $\frac{1}{2}$ nine times we have $\frac{9}{18}$.

12. Change the following to 18ths, stating the equations only: $\frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \frac{2}{3}, \frac{5}{9}, \frac{7}{9}, \frac{8}{9}, 2\frac{1}{2}, 1\frac{1}{3}, 3\frac{1}{6}$.

13. Reduce the following to 20ths: $\frac{1}{2}, \frac{1}{5}, \frac{1}{4}, \frac{1}{10}, \frac{2}{5}, \frac{3}{4}, \frac{2}{5}, \frac{7}{10}, \frac{9}{10}, 3\frac{1}{5}$.

14. Reduce the following to 24ths: $\frac{1}{3}, \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{12}, \frac{7}{24}, \frac{3}{8}, \frac{11}{24}, \frac{7}{6}$.

15. Reduce to 40ths: $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{1}{8}, \frac{1}{10}, \frac{1}{20}, \frac{3}{8}, \frac{9}{10}, \frac{4}{5}, 4\frac{3}{4}$.

16. Reduce to 100ths: $\frac{1}{20}, \frac{2}{25}, \frac{7}{10}, \frac{9}{50}, 1\frac{2}{5}, 2\frac{7}{10}, 3\frac{1}{20}, 4\frac{4}{25}, 5\frac{1}{50}, 4\frac{3}{5}$.

LESSON VII.

Reduction to Common Denominator.

When two or more fractions are given like denominators, they are said to have a *Common Denominator*.

The *Least Common Denominator* is the least number that can be divided by all the denominators of the given fractions.

1. Reduce the following to equivalent fractions having a common denominator: $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$.

Solution.—The least number that can be exactly divided by the denominators 3, 4, and 6, is 12. By reducing each fraction to 12ths, we obtain $\frac{8}{12}, \frac{9}{12}$, and $\frac{10}{12}$.

2. Reduce the following to their least common denominator:

1. $\frac{3}{4}, \frac{1}{2}$.

4. $\frac{3}{7}, \frac{1}{2}$.

7. $\frac{5}{6}, \frac{5}{8}$.

10. $\frac{11}{14}, \frac{5}{7}, \frac{9}{8}$.

2. $\frac{2}{3}, \frac{5}{6}$.

5. $\frac{2}{3}, \frac{4}{5}$.

8. $\frac{7}{9}, \frac{3}{4}$.

11. $\frac{13}{8}, \frac{5}{6}, \frac{7}{12}$.

3. $\frac{7}{8}, \frac{3}{4}$.

6. $\frac{7}{8}, \frac{3}{5}$.

9. $\frac{9}{10}, \frac{3}{5}, \frac{7}{30}$.

12. $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$.

3. Reduce to their least common denominator:

1. $\frac{5}{8}, \frac{7}{32}, \frac{9}{16}, \frac{3}{4}$.

6. $\frac{3}{4}, \frac{11}{16}, \frac{9}{14}, \frac{1}{2}, \frac{5}{8}$.

2. $\frac{7}{9}, \frac{5}{6}, \frac{3}{8}, \frac{1}{18}$.

7. $\frac{1}{4}, \frac{3}{2}, \frac{9}{24}, \frac{7}{12}, \frac{5}{8}$.

3. $\frac{8}{15}, \frac{3}{5}, \frac{7}{9}, \frac{11}{12}$.

8. $\frac{9}{22}, \frac{1}{2}, \frac{29}{44}, \frac{6}{11}, \frac{1}{4}$.

4. $\frac{5}{16}, \frac{7}{8}, \frac{2}{3}, \frac{5}{6}$.

9. $\frac{8}{15}, \frac{5}{6}, \frac{3}{20}, \frac{7}{8}, \frac{3}{10}$.

5. $\frac{7}{12}, \frac{3}{8}, \frac{5}{16}, \frac{9}{32}$.

10. $\frac{7}{24}, \frac{1}{8}, \frac{1}{12}, \frac{3}{16}, \frac{7}{8}$.

4. A dressmaker cut $\frac{1}{2}$ of a piece of silk into equal parts. If each part was $\frac{1}{12}$ of the whole piece, how many parts were there?

5. A charitable society gave to some poor persons $2\frac{2}{3}$ barrels of flour in equal portions. If each portion was $\frac{1}{4}$ of a barrel, how many persons were there?

6. If each of the above portions was 28 pounds, how many pounds of flour make a barrel?

7. If you have $\frac{1}{8}$ of a dollar, how many apples at $\frac{1}{20}$ of a dollar each can you buy therewith?

8. A field is $\frac{1}{16}$ of an entire farm. If the field be cut into equal lots, each the $\frac{1}{96}$ of the farm, how many lots will be made?

LESSON VIII.**Reduction to Lower Terms.**

1. How many eighths in $\frac{15}{4}$?

Solution.—1. The equations are $\frac{1}{8} = \frac{3}{24}$ and $\frac{15}{4} = \frac{5}{8}$.

2. Since $\frac{1}{8} = \frac{3}{24}$, $\frac{15}{4} =$ as many 8ths as 3 is contained times in 15, which is 5 times.

3. Therefore, $\frac{15}{4} = \frac{5}{8}$.

2. In like manner reduce the following to halves: $\frac{6}{13}$, $\frac{4}{8}, \frac{30}{60}, \frac{8}{16}, \frac{16}{32}, \frac{18}{36}, \frac{19}{38}, \frac{25}{50}, \frac{11}{22}, \frac{14}{28}$.

3. Reduce the following to thirds: $\frac{18}{27}$, $\frac{11}{33}$, $\frac{16}{48}$, $\frac{28}{42}$, $\frac{18}{54}$, $\frac{26}{39}$, $\frac{92}{138}$, $\frac{13}{39}$, $\frac{84}{126}$.
4. Reduce to fourths: $\frac{33}{44}$, $\frac{24}{48}$, $\frac{51}{68}$, $\frac{36}{72}$, $\frac{63}{84}$, $\frac{24}{96}$, $\frac{75}{100}$, $\frac{48}{64}$, $\frac{28}{56}$, $\frac{26}{62}$.
5. Since $\frac{1}{12} = \frac{1}{2}$, $\frac{9}{27} = \frac{1}{3}$, etc., does the change thus made in the terms of the fraction change its value?
6. Does dividing both terms of a fraction by the same number change its value?
7. By thus dividing change $\frac{18}{54}$ to thirds.
 1. The equations are $18 \div 3 = 6$, and $\frac{18}{54} = \frac{1}{3}$.
 2. $\frac{1}{3}$ of $18 = 6$, and dividing both terms of $\frac{18}{54}$ by 6 we have $\frac{1}{3}$.
 3. Therefore, $\frac{18}{54} = \frac{1}{3}$.
8. Reduce the following fractions to lower terms: $\frac{3}{6}$, $\frac{5}{25}$, $\frac{9}{12}$, $\frac{8}{16}$, $\frac{15}{20}$, $\frac{16}{32}$, $\frac{22}{88}$, $\frac{27}{86}$, $\frac{19}{38}$, $\frac{33}{44}$.
9. Reduce the following fractions to *lowest* terms:
 1. $\frac{16}{48}$, $\frac{12}{48}$, $\frac{25}{50}$, $\frac{14}{52}$, $\frac{18}{64}$, $\frac{28}{68}$, $\frac{17}{72}$, $\frac{18}{72}$, $\frac{11}{12}$, $\frac{26}{36}$.
 2. $\frac{52}{65}$, $\frac{65}{68}$, $\frac{13}{91}$, $\frac{14}{28}$, $\frac{42}{46}$, $\frac{84}{98}$, $\frac{28}{42}$, $\frac{42}{56}$, $\frac{30}{60}$, $\frac{45}{60}$.
 3. $\frac{12}{84}$, $\frac{28}{84}$, $\frac{56}{84}$, $\frac{70}{84}$, $\frac{42}{84}$, $\frac{12}{96}$, $\frac{32}{96}$, $\frac{64}{96}$, $\frac{18}{96}$, $\frac{72}{96}$.
 4. $\frac{48}{72}$, $\frac{80}{96}$, $\frac{32}{64}$, $\frac{16}{80}$, $\frac{5}{45}$, $\frac{39}{78}$, $\frac{70}{98}$, $\frac{15}{30}$, $\frac{14}{84}$, $\frac{24}{96}$.
 5. $\frac{84}{96}$, $\frac{46}{92}$, $\frac{18}{39}$, $\frac{132}{144}$, $\frac{77}{121}$, $\frac{9}{27}$, $\frac{16}{64}$, $\frac{25}{125}$, $\frac{36}{216}$, $\frac{49}{343}$.

Problems.

1. My coat cost me $\frac{2}{3}$ of \$63. What did I pay for it?

Solution.—1. The equations are $\frac{2}{3} = \frac{1}{3}$, and $\frac{1}{3}$ of 63 = 21.
 2. Reducing $\frac{2}{3}$ to its lowest terms I have $\frac{1}{3}$.
 3. Taking $\frac{1}{3}$ of \$63, I have \$21.
 4. Therefore, my coat cost me \$21.

2. If a violin cost me $\frac{16}{36}$ of \$81, how much did I pay for it?

3. If a man bought a colt for \$55 and sold it for $\frac{1}{2}\frac{1}{2}$ of the cost, what did he receive for it?

4. A boy having 84 marbles lost $\frac{8}{12}$ of them; he then bought $\frac{6}{14}$ as many as remained. How many did he then have?

LESSON IX.

Reduction of Mixed Numbers.

1. How many thirds in $5\frac{2}{3}$?

Solution.—In 1 there are $\frac{3}{3}$; hence in 5 there are 5 times $\frac{3}{3}$, or $1\frac{5}{3}$. $1\frac{5}{3} + \frac{2}{3} = 1\frac{7}{3}$. Therefore, $5\frac{2}{3} = 1\frac{7}{3}$.

2. Reduce to fourths: $8\frac{1}{4}$, $2\frac{1}{4}$, $8\frac{3}{4}$, $6\frac{3}{4}$, $3\frac{1}{4}$, $4\frac{1}{4}$, $9\frac{1}{4}$, $12\frac{1}{4}$, 10 , $7\frac{1}{4}$.

3. Reduce to fifths: 5, $9\frac{3}{5}$, $5\frac{2}{5}$, $4\frac{1}{5}$, $6\frac{1}{5}$, $3\frac{3}{5}$, $4\frac{4}{5}$, $6\frac{3}{5}$, $5\frac{4}{5}$, $8\frac{4}{5}$.

4. Since $9\frac{3}{5} = 4\frac{5}{5} + \frac{3}{5}$, or $4\frac{8}{5}$, how can you very briefly obtain 48?

5. By simply multiplying and adding, reduce the following to improper fractions: $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{2}{3}$, $4\frac{1}{4}$, $3\frac{1}{5}$, $5\frac{1}{6}$, $6\frac{1}{4}$, $8\frac{1}{2}$.

6. Reduce to improper fractions: $4\frac{1}{5}$, $5\frac{3}{7}$, $6\frac{5}{6}$, $7\frac{2}{5}$, $3\frac{7}{6}$, $4\frac{2}{3}$, $6\frac{5}{7}$, $7\frac{1}{4}$, $8\frac{1}{6}$, $6\frac{2}{5}$, $5\frac{3}{8}$, $7\frac{5}{6}$, $8\frac{7}{9}$, $9\frac{2}{5}$, $6\frac{7}{8}$, $8\frac{2}{3}$, $6\frac{1}{2}$, $9\frac{3}{8}$.

7. Reduce to improper fractions: $8\frac{1}{6}$, $6\frac{3}{5}$, $5\frac{2}{3}$, $7\frac{5}{6}$, $8\frac{5}{7}$, $9\frac{2}{5}$, $10\frac{5}{7}$, $11\frac{4}{5}$, $6\frac{7}{8}$, $6\frac{1}{5}$, $10\frac{5}{8}$, $11\frac{2}{3}$, $12\frac{7}{8}$.

Reduction of Improper Fractions.

1. Reduce $3\frac{7}{6}$ to a mixed number.

Solution.—There are $\frac{6}{6}$ in 1; hence $3\frac{7}{6} =$ as many units as $\frac{6}{6}$ is contained times in $3\frac{7}{6}$, or $6\frac{1}{6}$. Therefore, $3\frac{7}{6} = 6\frac{1}{6}$.

2. Reduce to mixed numbers the following: $\frac{25}{4}$, $\frac{33}{5}$, $\frac{21}{4}$, $\frac{38}{6}$, $\frac{42}{5}$, $\frac{52}{8}$, $\frac{39}{5}$, $\frac{65}{8}$, $\frac{43}{4}$, $\frac{55}{6}$.

3. Reduce to mixed numbers: $\frac{44}{8}$, $\frac{48}{5}$, $\frac{64}{10}$, $\frac{78}{8}$, $\frac{88}{9}$, $\frac{68}{8}$, $\frac{71}{9}$, $\frac{77}{6}$, $\frac{94}{8}$, $\frac{93}{9}$.
4. Reduce to mixed numbers: $\frac{56}{6}$, $\frac{100}{11}$, $\frac{88}{9}$, $\frac{87}{9}$, $\frac{99}{10}$, $\frac{122}{11}$, $\frac{130}{20}$, $\frac{104}{25}$, $\frac{190}{20}$; $\frac{144}{11}$.
5. One dozen melons cut into tenths, and sold at the rate of 5 cents a tenth, will bring in how much money?
6. A student has $4\frac{3}{4}$ pages to learn. If he learns $\frac{1}{5}$ of a page every 5 minutes, how long will it take him to learn the whole lesson?
7. If a man mows $\frac{1}{8}$ of an acre of grass in $\frac{1}{2}$ an hour, how long will it take him to mow $7\frac{1}{8}$ acres?
8. From New York to Philadelphia is 90 miles. If a man walks $\frac{1}{20}$ of the distance in $1\frac{1}{2}$ hours, how many miles does he walk in $\frac{1}{2}$ an hour?
9. What is the excess of the 16ths over the 20ths, in 800?
10. I bought $8\frac{3}{4}$ quarts of wine at \$1 per quart. I sold it at 10 cents a gill ($\frac{1}{8}$ of a quart). How much did I lose?
11. The time of how many days will be gained from slumber in the month of April, by rising each morning $\frac{1}{2}$ of an hour earlier?
-

LESSON X.

Addition of Fractions.

- Find the halves and units in $\frac{1}{2} + \frac{1}{2}$. In $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$.
- How many thirds and units in $\frac{2}{3} + \frac{2}{3} + \frac{2}{3}$? In $\frac{2}{3} + \frac{2}{3} + \frac{1}{3}$?
- Find the sum of $\frac{1}{4} + \frac{2}{4} + \frac{1}{4}$. Of $\frac{3}{4} + \frac{1}{4} + \frac{2}{4}$. Of $\frac{4}{4} + \frac{1}{4} + \frac{5}{4} + \frac{2}{4}$.

4. What is the sum of $\frac{1}{6}$, $\frac{2}{3}$ and $\frac{3}{5}$? Of $\frac{5}{6}$, $\frac{2}{3}$ and $\frac{3}{5}$? Of $\frac{1}{6}$ and $\frac{3}{5}$?

5. What is the sum of $\frac{3}{4}$ and $\frac{1}{2}$?

Solution.—Fourths and halves, as such, cannot be added; and, therefore, they must be reduced to equivalent fractions having a common denominator. $\frac{1}{4} = \frac{2}{8}$. $\frac{3}{4} + \frac{2}{8} = \frac{5}{8} = 1\frac{1}{4}$. Therefore, the sum of $\frac{3}{4}$ and $\frac{1}{2}$ is $1\frac{1}{4}$.

6. Find the value of $\frac{1}{2} + \frac{1}{3}$. Of $\frac{1}{3} + \frac{1}{6}$. Of $\frac{1}{6} + \frac{1}{2}$. Of $\frac{1}{2} + \frac{5}{6}$.

7. What is the sum of $\frac{1}{2}$ and $\frac{1}{3}$? Of $\frac{1}{2}$ and $\frac{1}{6}$? Of $\frac{1}{2}$ and $\frac{1}{5}$?

8. Reduce and add $\frac{1}{2}$ and $\frac{1}{6}$; $\frac{1}{3}$ and $\frac{1}{6}$; $\frac{3}{4}$ and $\frac{2}{3}$; $\frac{2}{3}$ and $\frac{9}{10}$.

9. Find the value of $\frac{8}{9} + \frac{1}{18}$; of $\frac{7}{6}$ and $\frac{5}{12}$; of $\frac{9}{16}$ and $\frac{3}{8}$; of $\frac{5}{9}$ and $\frac{5}{18}$.

10. Reduce and add $\frac{1}{2}$ and $\frac{1}{3}$; $\frac{1}{3}$ and $\frac{1}{4}$; $\frac{1}{4}$ and $\frac{1}{5}$; $\frac{1}{5}$ and $\frac{1}{6}$.

11. Find the value of:

$$1. \frac{1}{2} + \frac{2}{3} + \frac{1}{6}.$$

$$9. \frac{5}{8} + \frac{7}{5} + \frac{11}{12}.$$

$$2. \frac{1}{4} + \frac{5}{6} + \frac{1}{12}.$$

$$10. \frac{2}{3} + \frac{5}{7} + \frac{7}{9}.$$

$$3. \frac{3}{4} + \frac{1}{6} + \frac{11}{12}.$$

$$11. \frac{1}{6} + \frac{1}{2} + \frac{1}{12}.$$

$$4. \frac{2}{3} + \frac{1}{15} + \frac{3}{5}.$$

$$12. \frac{2}{3} + \frac{1}{15} + \frac{3}{5}.$$

$$5. \frac{1}{3} + \frac{2}{3} + \frac{3}{4}.$$

$$13. \frac{11}{12} + \frac{1}{6} + \frac{1}{3}.$$

$$6. \frac{3}{4} + \frac{4}{5} + \frac{9}{10}.$$

$$14. \frac{5}{6} + \frac{7}{8} + \frac{2}{3}.$$

$$7. \frac{5}{6} + \frac{7}{9} + \frac{11}{12}.$$

$$15. \frac{1}{5} + \frac{1}{4} + \frac{9}{20}.$$

$$8. \frac{6}{7} + \frac{7}{8} + \frac{5}{56}.$$

$$16. \frac{17}{20} + \frac{3}{4} + \frac{1}{3}.$$

12. A man, in settling some petty bills, paid $\frac{1}{6}$ of a dollar to a porter, $\frac{2}{5}$ to a baker, $\frac{3}{8}$ to a grocer, and $\frac{4}{9}$ to his cook. How much money did he expend?

13. If I spend $\frac{1}{4}$ of a dollar for breakfast and $\frac{2}{5}$ for dinner, how much do I spend for both?

14. If a boy spent $\frac{1}{5}$ of his money on Monday, $\frac{3}{10}$ on Tuesday, and $\frac{7}{15}$ on Wednesday, what part did he spend?

LESSON XI.

1. Find the sum of $\frac{5}{6}$ and $\frac{3}{4}$.

Solution.—1. The least common denominator is 12.

2. $\frac{5}{6} = \frac{10}{12}$; $\frac{3}{4} = \frac{9}{12}$.

3. $\frac{10}{12} + \frac{9}{12} = \frac{19}{12} = 1\frac{7}{12}$.

4. Therefore, the sum of $\frac{5}{6}$ and $\frac{3}{4} = 1\frac{7}{12}$.

2. Find the value of the following:

1. $\frac{1}{5} + \frac{1}{12}$.

6. $\frac{1}{10} + \frac{1}{15}$.

11. $\frac{2}{3} + \frac{4}{5}$.

2. $\frac{1}{2} + \frac{4}{5}$.

7. $\frac{3}{5} + \frac{3}{4}$.

12. $\frac{4}{11} + \frac{11}{4}$.

3. $\frac{1}{7} + \frac{1}{5}$.

8. $\frac{2}{5} + \frac{2}{3}$.

13. $\frac{8}{15} + \frac{9}{25}$.

4. $\frac{1}{11} + \frac{1}{12}$.

9. $\frac{5}{6} + \frac{5}{9}$.

14. $\frac{5}{6} + \frac{7}{9} + \frac{11}{12}$.

5. $\frac{4}{5} + \frac{1}{4}$.

10. $\frac{7}{10} + \frac{8}{15}$.

15. $\frac{2}{3} + \frac{7}{9} + \frac{4}{7}$.

3. What is the value of $3\frac{1}{2} + 2\frac{1}{3}$?

Solution.—1. The equations are $3 + 2 = 5$; $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$; $5 + \frac{5}{6} = 5\frac{5}{6}$.

2. The sum of the integers is $3 + 2$, or 5.

3. The sum of the fractions is $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.

4. The sum of integers and fractions is $5 + \frac{5}{6} = 5\frac{5}{6}$.

4. Find the value of the following:

1. $3\frac{1}{2} + 1\frac{1}{5}$.

6. $3\frac{7}{8} + 3\frac{1}{2}$.

2. $4\frac{1}{2} + 5\frac{2}{3}$.

7. $5\frac{1}{3} + 6\frac{1}{4}$.

3. $6\frac{1}{2} + 5\frac{1}{3}$.

8. $6\frac{2}{3} + 5\frac{1}{6}$.

4. $4\frac{5}{6} + 5\frac{2}{3}$.

9. $3\frac{1}{8} + 8\frac{3}{7}$.

5. $7\frac{5}{6} + 7\frac{2}{4} + 7\frac{1}{2}$.

10. $8\frac{2}{9} + 6\frac{1}{8} + 9\frac{1}{4}$.

5. A certain field contains $10\frac{3}{4}$ acres and an adjacent field $9\frac{1}{2}$ acres. How many acres in the two fields?

6. A lady, while shopping, bought $4\frac{1}{2}$ yards of lace, $4\frac{2}{3}$ yards of ribbon, and $4\frac{5}{6}$ yards of cloth. How many yards in all did she buy?

7. A bicycler rode on Monday $20\frac{1}{2}$ miles, on Tuesday $18\frac{1}{2}$ miles, and on Wednesday $15\frac{2}{3}$ miles. How many miles in the three days?

LESSON XII.

1. What is the sum of:

- | | | |
|---|--|--|
| 1. $\frac{1}{5}$ and $\frac{1}{9}$? | 6. $\frac{8}{15}$ and $\frac{7}{25}$? | 11. $\frac{6}{8}$ and $\frac{5}{9}$? |
| 2. $\frac{1}{9}$ and $\frac{1}{10}$? | 7. $\frac{1}{11}$ and $\frac{1}{12}$? | 12. $\frac{3}{4}$ and $\frac{4}{7}$? |
| 3. $\frac{1}{5}$ and $\frac{1}{12}$? | 8. $\frac{2}{3}$ and $\frac{3}{5}$? | 13. $\frac{6}{7}$ and $\frac{7}{11}$? |
| 4. $1\frac{1}{4}$ and $1\frac{1}{11}$? | 9. $\frac{3}{5}$ and $\frac{4}{7}$? | 14. $1\frac{1}{6}$ and $1\frac{9}{10}$? |
| 5. $\frac{5}{8}$ and $\frac{1}{12}$? | 10. $\frac{2}{3}$ and $\frac{3}{4}$? | 15. $\frac{7}{8}$ and $\frac{6}{7}$? |

2. What is the sum of:

- | | |
|---|--|
| 1. $12\frac{1}{4} + 4\frac{1}{2} + 3\frac{1}{3}$? | 4. $8\frac{5}{8} + 3\frac{3}{4} + 21\frac{1}{2}$? |
| 2. $4\frac{4}{5} + 13\frac{3}{4} + 10\frac{1}{2}$? | 5. $6\frac{4}{5} + 5\frac{5}{6} + 4\frac{1}{4}$? |
| 3. $6\frac{2}{3} + 7\frac{1}{6} + 8\frac{1}{4}$? | 6. $4\frac{5}{6} + 1\frac{8}{9} + 3\frac{1}{4}$? |

3. A real-estate agent bought at one time $\frac{7}{8}$ of an acre of land and at another time $\frac{4}{5}$ of an acre. How much land did he buy?

4. A teacher assigned $\frac{4}{5}$ of an hour for the study of grammar and $\frac{6}{5}$ of an hour for arithmetic. How much time was allowed for the two studies?

5. A farmer sold hay for $\$20\frac{3}{4}$, oats for $\$15\frac{2}{3}$, and wheat for $\$35\frac{7}{8}$. How much did he receive for all?

6. A girl gave away $\frac{5}{6}$ of her roses and kept $\frac{1}{6}$. If she kept 8 roses, how many had she at first? How many did she give away?

7. If $(\frac{1}{3} + \frac{1}{4} + \frac{1}{6})$ of a number equals 1800, what is the number?

SUGGESTION.—1. $\frac{1}{3} + \frac{1}{4} + \frac{1}{6} =$ what?

2. How many 12ths of the number equal 1800?

3. How many 12ths are there in the number?

8. Two-thirds of a pole is in the air, $\frac{1}{3}$ in water, and the rest in mud. If 50 feet of it is in air and water, what is the length of the pole?

9. A colt cost me \$60, and $\frac{2}{3}$ of the cost of the colt is $\frac{1}{2}$ the cost of my horse. Find the cost of my horse.

10. A gas-range consumed in June $4\frac{1}{2}$ thousand feet, in July $3\frac{1}{2}$ thousand feet, and in August $2\frac{1}{2}$ thousand feet. Find the number of thousand feet consumed in the three months, and the cost at \$1.00 per thousand feet.

11. A seamstress cut from a piece of silk at one time $5\frac{3}{4}$ yards and at another time $7\frac{1}{4}$ yards, leaving $10\frac{1}{2}$ yards. How many yards were in the piece at first?

LESSON XIII.

Subtraction of Fractions.

1. Find the difference between these: $\frac{3}{5}$ and $\frac{2}{3}$; $\frac{5}{4}$ and $\frac{3}{4}$; $\frac{7}{8}$ and $\frac{5}{6}$; $\frac{7}{11}$ and $\frac{4}{11}$; $\frac{9}{12}$ and $\frac{5}{12}$.

2. Subtract $\frac{2}{3}$ from $\frac{4}{5}$; $\frac{1}{3}$ from $\frac{1}{2}$; $\frac{1}{5}$ from $\frac{1}{4}$; $\frac{1}{4}$ from $\frac{1}{3}$; $\frac{1}{6}$ from $\frac{1}{5}$.

3. Find the value of $\frac{11}{4} - \frac{4}{11}$; $\frac{7}{8} - \frac{1}{12}$; $\frac{8}{5} - \frac{7}{25}$; $\frac{11}{11} - \frac{1}{12}$; $\frac{2}{3} - \frac{3}{5}$; $\frac{3}{5} - \frac{4}{3}$.

4. What is the difference between $\frac{2}{3}$ and $\frac{3}{5}$? $\frac{5}{6}$ and $\frac{1}{4}$? $\frac{4}{5}$ and $\frac{2}{3}$? $\frac{5}{8}$ and $\frac{5}{6}$? $\frac{3}{4}$ and $\frac{4}{5}$?

5. Subtract $\frac{1}{8}$ from $\frac{1}{2}$; $\frac{4}{9}$ from $\frac{3}{5}$; $\frac{7}{11}$ from $\frac{2}{3}$; $\frac{11}{12}$ from $\frac{11}{11}$.
 6. What is the value of $\frac{7}{10} - \frac{5}{8}$? $\frac{7}{10} - \frac{3}{8}$? $\frac{11}{8} - \frac{7}{8}$?
 $\frac{7}{10} - \frac{9}{16}$? $\frac{9}{15} - \frac{9}{27}$?
 7. Subtract as indicated: $\frac{13}{16} - \frac{5}{8}$; $\frac{5}{7} - \frac{3}{5}$; $\frac{5}{16} - \frac{3}{24}$;
 $\frac{4}{16} - \frac{7}{32}$; $\frac{19}{5} - \frac{3}{4}$; $\frac{12}{21} - \frac{7}{49}$.
- | | | | | | | | | | |
|--------------------|---------------|---------------|---------------|---------------|---------------|----------------|----------------|----------------|----------------|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| $\frac{5}{6}$ | $\frac{4}{5}$ | $\frac{2}{3}$ | $\frac{3}{4}$ | $\frac{8}{9}$ | $\frac{6}{7}$ | $\frac{1}{5}$ | $\frac{8}{15}$ | $\frac{11}{5}$ | $\frac{1}{10}$ |
| Take $\frac{4}{5}$ | $\frac{3}{7}$ | $\frac{4}{9}$ | $\frac{3}{5}$ | $\frac{2}{3}$ | $\frac{3}{4}$ | $\frac{1}{12}$ | $\frac{7}{25}$ | $\frac{5}{11}$ | $\frac{1}{11}$ |
8. From $\frac{5}{6}$ $\frac{4}{5}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{8}{9}$ $\frac{6}{7}$ $\frac{1}{5}$ $\frac{8}{15}$ $\frac{11}{5}$ $\frac{1}{10}$.
 Take $\frac{4}{5}$ $\frac{3}{7}$ $\frac{4}{9}$ $\frac{3}{5}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{1}{12}$ $\frac{7}{25}$ $\frac{5}{11}$ $\frac{1}{11}$.
9. Perform the operation indicated in these:
- | | | |
|------------------------------------|------------------------------------|---------------------------------------|
| 1. $\frac{13}{16} - \frac{3}{4}$. | 6. $\frac{23}{30} - \frac{5}{6}$. | 11. $\frac{13}{14} - \frac{6}{7}$. |
| 2. $\frac{17}{18} - \frac{5}{9}$. | 7. $\frac{29}{21} - \frac{6}{7}$. | 12. $\frac{21}{22} - \frac{1}{2}$. |
| 3. $\frac{19}{20} - \frac{3}{4}$. | 8. $\frac{25}{36} - \frac{5}{6}$. | 13. $\frac{39}{40} - \frac{5}{8}$. |
| 4. $\frac{11}{12} - \frac{3}{5}$. | 9. $\frac{5}{8} - \frac{4}{7}$. | 14. $\frac{51}{55} - \frac{3}{5}$. |
| 5. $\frac{33}{40} - \frac{3}{4}$. | 10. $\frac{7}{9} - \frac{7}{10}$. | 15. $\frac{26}{28} - \frac{13}{18}$. |

10. If you own $\frac{2}{3}$ of a ship and sell $\frac{1}{3}$ of the ship, how much do you still own?
-

LESSON XIV.

Addition and Subtraction.

1. A farmer has $\frac{1}{2}$ of his cattle in one field, $\frac{1}{3}$ of them in a second field, and the remainder in a third field. What part of them is in the third field?
2. If there are 28 cattle in the second field, how many more cattle has the farmer in the second field than in the first?
3. A man, being asked his age, said, "If you subtract 10 years from my age, you will have $\frac{4}{5}$ of my age." How old was he?
4. I bought gloves for $\frac{4}{5}$ of a dollar. I paid with a

two-dollar bill. How many fifths of a dollar did I receive in change? How many cents?

5. A man bought $\frac{2}{3}$ of a farm, added thereto $\frac{1}{2}$ more, and then sold a part equal to $\frac{1}{3}$ of the farm. What remained to him was worth \$2100. What was the value of the farm?

6. Of a bookseller's collection $\frac{1}{6}$ are maps, $\frac{3}{5}$ are books, and the rest periodicals. The periodicals are what part of the collection?

7. I bought a bicycle for $\frac{5}{6}$ of a certain sum of money and sold it for $\frac{7}{6}$ of that sum. How much did I gain? How much did I gain if the cost was \$112?

8. A girl earned 96 cents, spent $\frac{3}{4}$ of them, and again earned $\frac{2}{3}$ as many as she had at first. How much money did she then have?

9. A man bought a carriage for \$300 and paid $\frac{2}{3}$ as much for a sleigh. He sold the two for $\frac{6}{5}$ of their cost. How much did he lose by the sale?

10. A man has $\frac{1}{2}$ of his money in his safe, $\frac{1}{3}$ of it in his pocket, and the rest in bank. What part is in bank?

11. If his entire sum of money is \$7200, how many dollars has he in bank?

12. What fraction added to the sum of $\frac{1}{5}$, $\frac{3}{50}$, $\frac{9}{100}$, and $\frac{7}{20}$ will make $\frac{4}{5}$?

13. Find the value of:

$$1. \frac{1}{3} + \frac{1}{4} - \frac{1}{2}.$$

$$7. \frac{2}{3} + \frac{1}{2} - \frac{3}{4}.$$

$$13. \frac{3}{4} + \frac{8}{10} - \frac{4}{5}.$$

$$2. \frac{1}{3} + \frac{1}{4} - \frac{1}{6}.$$

$$8. \frac{4}{5} + \frac{1}{2} - \frac{2}{3}.$$

$$14. \frac{3}{5} - \frac{1}{10} + \frac{2}{15}.$$

$$3. \frac{1}{2} + \frac{1}{3} - \frac{1}{4}.$$

$$9. \frac{2}{3} + \frac{2}{5} - \frac{1}{2}.$$

$$15. \frac{3}{5} + \frac{3}{8} - \frac{5}{12}.$$

$$4. \frac{2}{3} + \frac{1}{2} - \frac{3}{4}.$$

$$10. \frac{5}{6} - \frac{4}{3} + \frac{1}{2}.$$

$$16. \frac{5}{8} - \frac{1}{3} + \frac{5}{12}.$$

$$5. \frac{1}{2} + \frac{1}{4} - \frac{1}{3}.$$

$$11. \frac{2}{3} + \frac{2}{5} - \frac{1}{2}.$$

$$17. \frac{5}{16} + \frac{3}{8} - \frac{1}{4}.$$

$$6. \frac{3}{4} + \frac{4}{5} - \frac{9}{10}.$$

$$12. \frac{7}{8} + \frac{5}{12} - \frac{3}{4}.$$

$$18. \frac{7}{8} - \frac{4}{5} + \frac{3}{10}.$$

LESSON XV.

1. What is the value of $6\frac{2}{3} - 4\frac{4}{5}$?

Solution.—1. $6\frac{2}{3} = 6\frac{10}{15}$; $4\frac{4}{5} = 4\frac{12}{15}$.

2. $6\frac{10}{15} - 4\frac{12}{15} = 5\frac{5}{15} - 4\frac{12}{15} = 1\frac{13}{15}$.

3. Since $\frac{1}{5} - \frac{12}{15}$ indicates an impossibility, we add one of the 6 units, that is $\frac{1}{5}$, to $\frac{1}{5}$, making $\frac{2}{5}$.

Note.—In both Addition and Subtraction it is considered best to operate with the integers and fractions separately.

2. Find the value of:

1. $7\frac{1}{4} - 4\frac{1}{2}$.

11. $5\frac{3}{4} - 3\frac{1}{4}$.

21. $5\frac{1}{10} - 4\frac{1}{2}$.

2. $5\frac{1}{2} - 4\frac{1}{2}$.

12. $4\frac{1}{5} - 2\frac{3}{10}$.

22. $4\frac{5}{8} - 2\frac{1}{4}$.

3. $4\frac{1}{4} - 3\frac{1}{3}$.

13. $4\frac{3}{4} - 3\frac{1}{6}$.

23. $5\frac{3}{8} - 3\frac{1}{4}$.

4. $4\frac{1}{2} - 2\frac{2}{5}$.

14. $4\frac{2}{5} - 2\frac{1}{4}$.

24. $6\frac{1}{4} - 4\frac{5}{8}$.

5. $6\frac{3}{4} - 2\frac{5}{8}$.

15. $9\frac{1}{8} - 3\frac{1}{4}$.

25. $5\frac{5}{6} - 3\frac{3}{10}$.

6. $7\frac{1}{8} - 4\frac{3}{4}$.

16. $8\frac{2}{3} - 4\frac{1}{6}$.

26. $6\frac{3}{8} - 3\frac{1}{4}$.

7. $9\frac{4}{5} - 8\frac{1}{2}$.

17. $10\frac{5}{11} - 4\frac{3}{4}$.

27. $11\frac{6}{7} - 1\frac{2}{3}$.

8. $12\frac{1}{2} - 9\frac{7}{8}$.

18. $13\frac{1}{4} - 6\frac{2}{3}$.

28. $14\frac{1}{6} - 7\frac{2}{3}$.

9. $15\frac{3}{4} - 11\frac{4}{5}$.

19. $16\frac{4}{5} - 10\frac{5}{6}$.

29. $17\frac{2}{3} - 12\frac{2}{3}$.

10. $18\frac{1}{4} - 7\frac{1}{3}$.

20. $19\frac{5}{8} - 8\frac{3}{4}$.

30. $20\frac{1}{8} - 8\frac{7}{8}$.

3. Find the value of:

1. $23\frac{1}{2} - 11\frac{3}{4}$.

4. $32 - 7\frac{5}{8}$.

7. $100 - 9\frac{1}{2}$.

2. $26\frac{2}{3} - 14\frac{7}{8}$.

5. $40 - 9\frac{7}{10}$.

8. $88 - 8\frac{7}{8}$.

3. $29 - 13\frac{1}{11}$.

6. $41 - 1\frac{1}{9}$.

9. $77 - 7\frac{6}{7}$.

4. Find the value of:

1. $8\frac{5}{6} - 3\frac{1}{2} + 5\frac{1}{3}$.

7. $8\frac{5}{6} - 3\frac{5}{12} + 4\frac{1}{3}$.

2. $11\frac{7}{9} + 4\frac{2}{3} - 5\frac{2}{9}$.

8. $3\frac{7}{8} - 1\frac{3}{4} + 7\frac{1}{2}$.

3. $20\frac{1}{8} + 2\frac{3}{4} - 9\frac{2}{3}$.

9. $7\frac{3}{4} - 3\frac{7}{16} + 2\frac{1}{8}$.

4. $7\frac{1}{6} + 1\frac{3}{10} - 3\frac{1}{20}$.

10. $9\frac{5}{8} + 2\frac{1}{2} - 6\frac{5}{16}$.

5. $6\frac{2}{3} + 4\frac{1}{6} - 3\frac{1}{12}$.

11. $2\frac{1}{4} + 3\frac{1}{4} - 4\frac{1}{6}$.

6. $7\frac{3}{8} - 4\frac{1}{4} + 5\frac{5}{16}$.

12. $11\frac{5}{8} + 4\frac{1}{2} - 8\frac{5}{16}$.

LESSON XVI.

Compound Fractions.

1. What is $\frac{3}{4}$ of $\frac{10}{11}$?

Solution.—1. The process is $\frac{1}{4}$ of $\frac{10}{11} = \frac{10}{44}$ and $\frac{3}{4}$ of $\frac{10}{11} = \frac{30}{44} = \frac{15}{22}$.
 2. Since $\frac{1}{4}$ of $\frac{10}{11} = \frac{10}{44}$, $\frac{3}{4}$ of $\frac{10}{11} = 3$ times $\frac{10}{44} = \frac{30}{44} = \frac{15}{22}$.
 3. Therefore, $\frac{3}{4}$ of $\frac{10}{11} = \frac{15}{22}$.

2. Find the value of the following compound fractions:

1. $\frac{3}{4}$ of $\frac{4}{5}$.	5. $\frac{5}{6}$ of $\frac{6}{5}$.	9. $\frac{3}{4}$ of $\frac{8}{10}$.
2. $\frac{3}{4}$ of $1\frac{2}{3}$.	6. $\frac{5}{6}$ of $1\frac{8}{2}$.	10. $\frac{7}{8}$ of $\frac{8}{17}$.
3. $\frac{3}{4}$ of $1\frac{6}{4}$.	7. $\frac{3}{4}$ of $1\frac{6}{6}$.	11. $\frac{8}{9}$ of $\frac{45}{44}$.
4. $\frac{5}{6}$ of $1\frac{2}{3}$.	8. $\frac{3}{4}$ of $1\frac{6}{9}$.	12. $\frac{9}{14}$ of $\frac{28}{20}$.

NOTE.—When Mixed Numbers occur in Compound Fractions, they must be reduced to Improper Fractions.

3. Find the value of:

1. $\frac{3}{4}$ of $1\frac{1}{2}$.	6. $\frac{3}{5}$ of $3\frac{1}{3}$.	11. $\frac{5}{6}$ of $2\frac{2}{3}$.
2. $\frac{3}{4}$ of $2\frac{2}{5}$.	7. $\frac{3}{5}$ of $2\frac{3}{6}$.	12. $\frac{5}{6}$ of $4\frac{2}{4}$.
3. $\frac{3}{4}$ of $5\frac{1}{3}$.	8. $\frac{3}{5}$ of $3\frac{1}{3}$.	13. $\frac{5}{6}$ of $4\frac{4}{5}$.
4. $\frac{3}{4}$ of $3\frac{1}{5}$.	9. $\frac{3}{5}$ of $4\frac{1}{5}$.	14. $\frac{5}{6}$ of $10\frac{2}{7}$.
5. $\frac{3}{4}$ of $5\frac{2}{5}$.	10. $\frac{3}{5}$ of $6\frac{2}{3}$.	15. $\frac{5}{6}$ of $8\frac{2}{7}$.

4. A woman bought $5\frac{1}{2}$ pounds of sugar and immediately used $\frac{3}{8}$ of it. How many pounds did she use?

5. Sarah rode in her carriage $\frac{1}{2}$ of $10\frac{2}{3}$ miles per hour. What was her rate per hour?

6. A gentleman appropriated $62\frac{1}{2}$ dollars for a suit of clothing, paying $\frac{3}{4}$ of the sum for the coat. What was the cost of the coat?

LESSON XVII.

1. What is $\frac{1}{5}$ of $\frac{7}{8}$?

Solution.—1. The process is $\frac{1}{5}$ of $\frac{7}{8} = \frac{7}{40}$ and $\frac{4}{5}$ of $\frac{7}{8} = \frac{28}{40} = \frac{7}{10}$.

2. Since $\frac{1}{5}$ of $\frac{7}{8} = \frac{7}{40}$, $\frac{4}{5}$ of $\frac{7}{8} = 4$ times $\frac{7}{40}$, or $\frac{28}{40} = \frac{7}{10}$.
 3. Therefore, $\frac{4}{5}$ of $\frac{7}{8} = \frac{7}{10}$.

2. Find the value of:

- | | | |
|---------------------------------------|--|---|
| 1. $\frac{5}{6}$ of $\frac{7}{10}$. | 10. $\frac{6}{7}$ of $\frac{8}{9}$. | 19. $\frac{5}{8}$ of $9\frac{1}{5}$. |
| 2. $\frac{2}{3}$ of $\frac{5}{4}$. | 11. $\frac{4}{5}$ of $\frac{5}{6}$. | 20. $\frac{2}{3}$ of $1\frac{1}{3}$. |
| 3. $\frac{3}{5}$ of $\frac{7}{9}$. | 12. $\frac{5}{11}$ of $\frac{2}{3}$. | 21. $\frac{3}{4}$ of $1\frac{3}{4}$. |
| 4. $\frac{3}{4}$ of $\frac{3}{5}$. | 13. $\frac{9}{10}$ of $1\frac{1}{2}$. | 22. $\frac{4}{5}$ of $2\frac{1}{3}$. |
| 5. $\frac{9}{10}$ of $\frac{7}{11}$. | 14. $\frac{2}{3}$ of $\frac{5}{6}$. | 23. $\frac{5}{6}$ of $7\frac{1}{2}$. |
| 6. $\frac{4}{5}$ of $\frac{9}{7}$. | 15. $\frac{4}{5}$ of $\frac{3}{4}$. | 24. $\frac{7}{6}$ of $8\frac{3}{4}$. |
| 7. $\frac{3}{4}$ of $\frac{3}{5}$. | 16. $\frac{5}{6}$ of $4\frac{1}{4}$. | 25. $1\frac{1}{2}$ of $12\frac{3}{5}$. |
| 8. $\frac{5}{6}$ of $\frac{2}{3}$. | 17. $\frac{2}{3}$ of $6\frac{3}{8}$. | 26. $\frac{7}{8}$ of $11\frac{1}{5}$. |
| 9. $\frac{3}{4}$ of $\frac{7}{8}$. | 18. $\frac{5}{7}$ of $7\frac{1}{2}$. | 27. $\frac{6}{11}$ of $24\frac{3}{4}$. |

3. If a man earns in a week $12\frac{1}{2}$ dollars, and spends $\frac{2}{3}$ of what he earns, how much does he spend?

4. A lady bought $\frac{2}{3}$ of $37\frac{1}{2}$ yards of silk, paying therefore \$1.50 per yard. How much did the silk cost her?

5. I owned $\frac{1}{5}$ of a ship and sold $\frac{2}{3}$ of my share for \$21,000. What was the value of the ship?

6. A farmer sold a cow for \$37 $\frac{1}{2}$. If the cost was $\frac{2}{3}$ of the selling price, how much did the cow cost?

7. A bicycler, with the aid of the wind, rode $29\frac{3}{5}$ miles. If the wind carried him over $\frac{1}{37}$ of that distance, how far would he have ridden without the aid of the wind?

8. A gentleman had a collection of bric-a-brac that cost him \$396. He sold $\frac{3}{4}$ of the collection for what $\frac{3}{4}$ cost him. How much did he gain on the part sold?

9. A plum-tree, being shaken, yielded $\frac{4}{5}$ of a peck of plums. Two-fifths of them were sold at the rate of \$.50 a peck. How much money was received for them?

10. In a mixture of black and green tea, $\frac{5}{8}$ is green tea. Find the number of pounds of each kind, if the mixture weighs $14\frac{1}{2}$ pounds.

11. A man fell heir to $\frac{5}{6}$ of an amount of bank stock worth \$21,000. With $\frac{5}{6}$ of his fortune he bought 90 shares of railroad stock. How much did each share cost him?

LESSON XVIII.

Multiplication of Fractions.

1. Find 3 times each of these: $\frac{1}{3}, \frac{3}{4}, \frac{4}{7}, \frac{5}{8}, \frac{7}{10}, \frac{9}{11}, \frac{3}{4}, \frac{1}{2}, \frac{4}{5}, \frac{11}{12}$.
2. Find 4 times each of these: $\frac{1}{6}, \frac{2}{3}, \frac{6}{7}, \frac{3}{4}, \frac{5}{8}, \frac{6}{7}, \frac{8}{9}, \frac{9}{10}, \frac{11}{12}, \frac{11}{12}$.
3. Find 5 times these: $\frac{1}{7}, \frac{3}{4}, \frac{4}{5}, \frac{5}{8}, \frac{7}{11}, \frac{9}{10}, \frac{11}{12}, \frac{13}{15}, \frac{13}{15}, \frac{23}{20}$.
4. From $\frac{5}{6}$ of 7 take 7 times $\frac{5}{6}$, and state the difference.
5. Take $\frac{2}{3}$ of 6, 8, 10, 12, 15, 20, 25, 30.
6. Take 7 times each of the following: $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}, \frac{9}{10}, \frac{11}{12}, \frac{11}{12}, \frac{13}{15}, \frac{13}{15}$.
7. Multiply $5\frac{3}{4}$ by 6.

Solution.—1. The process is $5 \times 6 = 30$; $\frac{3}{4} \times 6 = \frac{18}{4}$ $= 4\frac{1}{2}$; $30 + 4\frac{1}{2} = 34\frac{1}{2}$.

2. Six times the integer 5 $= 30$; 6 times the fraction $\frac{3}{4} = \frac{18}{4} = 4\frac{1}{2}$.

3. 30, the first product, $+ 4\frac{1}{2}$, the second product, $= 34\frac{1}{2}$, the entire product.

4. Therefore, $5\frac{3}{4} \times 6 = 34\frac{1}{2}$.

8. Find the value of the following expressions: $5\frac{2}{3} \times 6$; $7\frac{3}{4} \times 5$; $5\frac{4}{7} \times 4$; $8\frac{4}{5} \times 6$; $7\frac{3}{5} \times 8$.

9. How much is 6 times $5\frac{2}{3}$? 5 times $7\frac{3}{4}$? 4 times $5\frac{4}{7}$? 6 times $8\frac{4}{5}$? 8 times $7\frac{3}{5}$?

10. Find the product of:

$$\begin{array}{r} 6\frac{4}{5} \\ 10\frac{1}{4} \\ 8\frac{2}{3} \\ 8\frac{5}{7} \\ 4\frac{4}{9} \\ \hline 6 \\ 5 \\ 7 \\ 8 \\ 9 \\ \hline \end{array} \quad \begin{array}{r} 8\frac{4}{5} \\ 9\frac{3}{7} \\ 4\frac{4}{6} \\ 6\frac{2}{5} \\ 3\frac{5}{6} \\ \hline 10 \\ 7 \\ 4 \\ 9 \\ 11 \\ \hline \end{array}$$

11. What is the product of:

- | | | |
|-------------------------------|---------------------------------|---------------------------------|
| 1. $3\frac{2}{5} \times 11$? | 6. $9\frac{2}{3} \times 8$? | 11. $9 \times 8\frac{2}{5}$? |
| 2. $5\frac{1}{3} \times 9$? | 7. $10\frac{1}{6} \times 9$? | 12. $10 \times 7\frac{5}{7}$? |
| 3. $4\frac{2}{3} \times 5$? | 8. $8\frac{3}{7} \times 7$? | 13. $11 \times 4\frac{6}{7}$? |
| 4. $7 \times 6\frac{1}{3}$? | 9. $10\frac{4}{11} \times 10$? | 14. $12 \times 5\frac{2}{9}$? |
| 5. $11\frac{2}{5} \times 2$? | 10. $7\frac{4}{5} \times 7$? | 15. $13 \times 2\frac{3}{10}$? |
-

LESSON XIX.

1. Victoria bought $10\frac{2}{3}$ yards of trimming at 16 cents per yard. How much did she pay for it?

2. A superintendent hired 8 men to work at $3\frac{1}{4}$ dollars a day. How much did the 8 men cost him daily?

3. Twelve times $6\frac{1}{2}$ years is twice my age. How old am I?

4. If a boy earns $10\frac{1}{2}$ cents an hour, how much will he earn in $\frac{5}{6}$ of 18 hours?
5. From Philadelphia to Trenton is 33 miles. How far is a man from Trenton who, starting in Philadelphia, has ridden 2 hours, at the rate of $10\frac{1}{2}$ miles per hour?
6. What will $10\frac{5}{6}$ cords of wood cost at \$7 per cord?
7. If a barrel of potatoes contains $2\frac{3}{4}$ bushels, what is the value of potatoes per barrel, at the rate of 50 cents per bushel?
8. If the railroad fare between two places is $\$7\frac{37}{100}$, what is the total fare of 10 persons?
9. If a coat cost \$15, and a pair of pantaloons and a vest cost $\frac{2}{3}$ as much, what did the whole suit cost?
10. Sarah used, in trimming hats, at one time $4\frac{2}{3}$ yards of ribbon and at another time $5\frac{2}{3}$ yards. Find the cost of the ribbon at 15 cents per yard.
11. I bought apples at $1\frac{1}{2}$ dollars per bushel, taking 24 bushels, and paying with a 50-dollar bill. How much change did I receive?
12. My friend and I started from the same place at the same time, and travelled in opposite directions, he at the rate of $4\frac{1}{2}$ miles per hour, and I at the rate of $4\frac{1}{3}$ miles per hour. How far apart were we at the end of 5 hours?
13. A man, keeping horses, bought 12 tons of hay at $\$15\frac{1}{2}$ per ton, and 100 bushels of corn at $\$.55\frac{1}{2}$. How much was the whole bill?
14. A householder provided as follows: $6\frac{2}{3}$ pounds of coffee at 28 cents and $5\frac{2}{3}$ pounds of butter at 24 cents. What was the amount of the bill?

LESSON XX.

1. How much is $4\frac{1}{3}$ times $5\frac{1}{4}$?

Solution.—1. The process is $4\frac{1}{3} = \frac{13}{3}$; $5\frac{1}{4} = \frac{21}{4}$. $\frac{13}{3}$ of $\frac{21}{4} = \frac{21}{4} = 22\frac{3}{4}$.

2. The mixed numbers reduced to improper fractions become $\frac{13}{3}$ and $\frac{21}{4}$.

3. $\frac{13}{3}$ of $\frac{21}{4} = 13$ times $\frac{1}{3}$ of $\frac{21}{4} = 13$ times $\frac{7}{4} = \frac{91}{4} = 22\frac{3}{4}$.

4. Therefore, $4\frac{1}{3}$ times $5\frac{1}{4} = 22\frac{3}{4}$.

2. Find the value of the following: $3\frac{1}{2}$ times $\frac{4}{5}$; $4\frac{1}{4}$ times $2\frac{1}{2}$; $9\frac{1}{2}$ times $\frac{3}{4}$; $5\frac{1}{2}$ times $3\frac{3}{4}$.

3. What is the value of $\frac{2}{5} \times 5\frac{1}{3}$? $\frac{4}{5} \times 3\frac{3}{4}$? $7\frac{1}{2} \times 4\frac{1}{2}$? $9\frac{1}{2} \times 3\frac{3}{4}$? $10\frac{1}{2} \times 9\frac{1}{2}$?

4. What is the product of $4\frac{1}{2}$ and $2\frac{1}{3}$? $5\frac{3}{5}$ and $3\frac{1}{2}$? $6\frac{2}{3}$ and $2\frac{1}{3}$? $12\frac{1}{2}$ and $1\frac{1}{2}$?

5. Find the value of:

- | | | |
|--|---|---|
| 1. $8\frac{1}{2} \times 7\frac{3}{4}$. | 7. $6\frac{1}{3} \times 3\frac{1}{5}$. | 13. $14\frac{3}{4} \times 2\frac{1}{10}$. |
| 2. $5\frac{2}{3} \times 2\frac{1}{2}$. | 8. $11\frac{2}{3} \times 4\frac{1}{2}$. | 14. $\frac{1}{3}$ of $\frac{3}{4}$ of 16. |
| 3. $12\frac{1}{3} \times 6\frac{1}{4}$. | 9. $16\frac{3}{4} \times 2\frac{1}{6}$. | 15. $\frac{3}{4}$ of $\frac{5}{8}$ of 27. |
| 4. $7\frac{1}{4} \times 2\frac{1}{5}$. | 10. $12\frac{2}{3} \times 2\frac{1}{6}$. | 16. $\frac{4}{7}$ of $\frac{7}{8}$ of $8\frac{2}{3}$. |
| 5. $9\frac{1}{3} \times 3\frac{3}{5}$. | 11. $10\frac{4}{5} \times 4\frac{1}{3}$. | 17. $\frac{7}{9}$ of $\frac{1}{2}$ of $10\frac{4}{5}$. |
| 6. $15\frac{2}{3} \times 5\frac{1}{2}$. | 12. $8\frac{1}{3} \times 3\frac{4}{5}$. | 18. $\frac{5}{11}$ of $\frac{2}{3}$ of 66. |

6. What will $5\frac{1}{2}$ tons of coal cost at \$ $5\frac{3}{4}$ a ton?

7. At $12\frac{1}{2}$ cents a dozen, what will 10 dozen pears cost?

8. If a fire-grate burns $1\frac{1}{2}$ cords of wood in a month, and if a cord of wood costs \$10, what will the wood for $4\frac{1}{2}$ months cost?

9. When hay is worth \$ $21\frac{3}{4}$ per ton, how much will $7\frac{7}{12}$ of a ton cost?

10. If a square foot of land is worth \$.16 $\frac{2}{3}$, what is the value of $10\frac{1}{2}$ square feet?

11. What cost $5\frac{1}{10}$ pounds of coffee at $33\frac{1}{3}$ cents a pound?

12. A gentleman bought 50 feet of garden hose at $\$.20\frac{1}{2}$ per foot. Find the total cost.

13. I bought $4\frac{1}{2}$ tons of hay at $\$16\frac{1}{2}$ per ton, and gave therefor my services at $\$1\frac{1}{2}$ per hour. How many hours of service were required?

SUGGESTION.—1 ton required how many hours?

14. $7\frac{1}{2} \times 5\frac{5}{6}$ = how many times 7?

15. A boy is $5\frac{1}{2}$ years old, and his father is $6\frac{1}{2}$ times as old. How old is his father?

16. I have a horse and cow in pasture. For the horse I pay $\$1\frac{1}{2}$ per day and for the cow $\$1\frac{1}{2}$ more. How much do I pay for both per week?

17. If my gas bill is $\$13\frac{1}{2}$ and my coal bill is $2\frac{2}{3}$ times as much, what is the sum of the two bills?

18. If a bushel of wheat weighs $1\frac{1}{2}$ as much as a bushel of oats, how many bushels of oats will weigh as much as $5\frac{1}{2}$ bushels of wheat?

19. A man owning $\frac{1}{4}$ of a ship worth \$21,000, sold $\frac{1}{8}$ of his share. How much money did he realize from the sale?

20. If your salary, which is $\$15\frac{2}{3}$ per week, be raised to $1\frac{1}{2}$ of that sum, what will you then receive per week?

21. If a man can mow $5\frac{1}{2}$ acres of grass in a day, how many acres can he mow in $6\frac{1}{2}$ days?

22. If land produces 50 bushels of corn per acre, how much corn will $9\frac{1}{2}$ acres produce?

23. If a field containing $12\frac{1}{4}$ acres is worth \$62 per acre, find the value of $\frac{1}{4}$ of the field?

24. What is the cost of $\frac{3}{5}$ of $6\frac{1}{4}$ tons of coal, at $\frac{2}{3}$ of \$7 $\frac{1}{2}$ per ton?

LESSON XXI.

Division of Fractions.

Exercises.

1. How many times is $\frac{1}{8}$ contained in 3?

Solution.—1. The process is $3 = \frac{24}{8}$; $\frac{24}{8} \div \frac{1}{8} = 4\frac{4}{5}$.

2. Or, $1 \div \frac{1}{8} = 8$; $3 \div \frac{1}{8} = 3 \times 8 = 24$; $3 \div \frac{1}{8} = \frac{24}{5} = 4\frac{4}{5}$.

3. Since $1 \div \frac{1}{8}$ gives 8, $3 \div \frac{1}{8}$ gives $3 \times 8 = 24$, and $3 \div \frac{1}{8}$ gives $\frac{1}{8}$ of 24, or $4\frac{4}{5}$.

2. How many times is $\frac{1}{3}$ contained in each of the following: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0?

3. How many times is $\frac{1}{3}$ contained in each of these: 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 30?

4. How many times is $\frac{2}{3}$ contained in the following: 1, 4, 5, 8, 10, 12, 14, 16, 18, 20?

5. How many times is $\frac{2}{3}$ contained in these: 3, 5, 7, 9, 11, 13, 15, 17, 19?

6. Divide 5 by each of these: $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}$.

7. Divide 6 by $\frac{3}{4}$; by $1\frac{1}{2}$; by $2\frac{1}{2}$; by $1\frac{1}{3}$; by $3\frac{3}{4}$.

Problems.

8. At \$ $\frac{1}{4}$ a bushel, how much corn can be bought for \$6.00?

9. At $\$ \frac{3}{4}$ each, how many books can be bought for \$18?
10. If a man burns $\frac{1}{2}$ of a ton of coal in a month, how long will 8 tons last?
11. If I earn $\frac{1}{5}$ of a dollar a day, how long will I be in earning \$21?
12. How many pounds of sugar at $6\frac{1}{2}$ cents a pound can be bought for half a dollar?

Exercises.

13. If you divide $\frac{2}{3}$ into 5 equal parts, what fraction do you obtain to express one part?

Solution.—1. The equation is $\frac{1}{5}$ of $\frac{2}{3} = \frac{2}{15}$.

2. To divide a number by 5 is to take $\frac{1}{5}$ of it; therefore, to divide $\frac{2}{3}$ by 5 is to find $\frac{1}{5}$ of $\frac{2}{3}$, which is $\frac{2}{15}$.

3. Therefore, $\frac{2}{3}$ divided by 5 = $\frac{2}{15}$.

14. How many times is 3 contained in each of the following: $\frac{9}{10}, \frac{3}{4}, \frac{7}{6}, \frac{11}{4}, \frac{13}{6}, \frac{9}{5}, \frac{15}{8}, \frac{17}{6}$?
15. Divide $\frac{8}{3}$ by each of these: 1, 2, 4, 6, 7, 8, 10, 11.
16. Divide $\frac{4}{3}$ by each of these: 3, 4, 6, 8, 10, 11, 12, 15.
17. Divide $2\frac{3}{4}$ by each of these: 9, 6, 11, 5, 4, 3, 2, 1.

Problems.

18. If 7 men do $\frac{5}{6}$ of a piece of work in a day, what fraction of the work can 1 man do in a day?
19. If 6 dozen eggs cost $\frac{5}{6}$ of a dollar, how much does 1 dozen cost?
20. If 6 bushels of wheat cost $5\frac{3}{4}$ dollars, what is the cost of 1 bushel?
21. A man bought 10 pieces of cloth for $6\frac{3}{4}$ dollars. How much did each piece cost?
22. $\$ \frac{3}{5}$ was divided equally among 6 boys. What part of a dollar did each receive? How many cents?

LESSON XXII.

Exercises.

1. Divide $\frac{7}{8}$ by $\frac{5}{6}$.

Solution.—1. The equations are $\frac{7}{8} = \frac{21}{24}$; $\frac{5}{6} = \frac{20}{24}$; $\frac{21}{24} \div \frac{20}{24} = 21 \div 20 = 1\frac{1}{20}$. That is, $\frac{7}{8}$ and $\frac{5}{6}$, reduced to their L. C. D., become 21 and 20 twenty-fourths, and $21 \div 20 = 1\frac{1}{20}$.

Or, 2. $\frac{7}{8} \div 1 = \frac{7}{8}$; $\frac{7}{8} \div \frac{1}{5} = \frac{42}{8}$; $\frac{7}{8} \div \frac{5}{6} = \frac{42}{8} \div \frac{20}{24} = \frac{21}{20} = 1\frac{1}{20}$. That is, $\frac{1}{5}$ is contained in $\frac{7}{8}$ six times as often as 1, and $\frac{5}{6}$ is contained in $\frac{7}{8}$ one-fifth as many times as $\frac{1}{5}$.

$$3. \text{ Therefore, } \frac{7}{8} \div \frac{5}{6} = 1\frac{1}{20}.$$

2. Divide $\frac{7}{8}$ by each of these: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}$.

3. Divide $\frac{5}{6}$ by each of these: $\frac{2}{3}, \frac{2}{5}, \frac{3}{7}, \frac{7}{8}, \frac{9}{10}, \frac{11}{12}$.

4. Find the quotients when the divisor is $\frac{5}{6}$ and the dividends are as follows: $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{7}{16}, \frac{11}{14}, \frac{4}{5}, \frac{6}{7}, \frac{2}{3}$.

5. The dividends are $3\frac{1}{2}, 5\frac{1}{2}, 8\frac{1}{2}, 4\frac{1}{2}, 7\frac{1}{2}, 9\frac{1}{2}, 6\frac{1}{2}$; the divisor is $\frac{1}{5}$. Find the quotients.

6. Divide $\frac{5}{6}$ by $\frac{1}{5}$; by $\frac{4}{5}$; by $\frac{11}{12}$; by $\frac{6}{5}$; by $\frac{5}{6}$; by $\frac{7}{8}$.

7. Divide $3\frac{1}{4}$ by $\frac{1}{5}$; by $2\frac{1}{4}$; by $10\frac{1}{8}$; by $3\frac{2}{3}$; by $3\frac{1}{6}$; by $4\frac{4}{5}$.

Problems.

8. At $\frac{2}{5}$ of a dollar a pound, how many pounds of coffee can be bought for $\frac{9}{10}$ of a dollar?

9. At $\$2\frac{1}{2}$ a pound, how many pounds of tea can be bought for $\$2\frac{2}{5}$?

10. If a bushel of barley costs $\$2\frac{2}{5}$, how many bushels can be bought for $\$10\frac{9}{10}$?

11. If a yard of muslin costs $\frac{2}{3}$ of a dime, how much can you buy for $\frac{5}{6}$ of a dime?

12. A farmer received $\$2\frac{3}{5}$ for $5\frac{1}{2}$ bushels of oats. How much was that a bushel?

Exercises.

13. Find the value of :

- | | | |
|---|--|--|
| 1. $\frac{1\frac{1}{9}}{9} \div 7.$ | 11. $\frac{8}{11} \div 9.$ | 21. $8 \div \frac{8}{9}.$ |
| 2. $\frac{1\frac{6}{7}}{7} \div 8.$ | 12. $2\frac{5}{7} \div 7.$ | 22. $2\frac{10}{11} \div \frac{11}{2}.$ |
| 3. $10\frac{5}{9} \div 5.$ | 13. $5\frac{4}{9} \div 8.$ | 23. $25 \div 4\frac{1}{4}.$ |
| 4. $75 \div \frac{5}{9}.$ | 14. $\frac{7}{8} \div \frac{4}{9}.$ | 24. $11 \div \frac{7}{8}.$ |
| 5. $\frac{1\frac{1}{9}}{7} \div \frac{2}{7}.$ | 15. $3\frac{4}{7} \div \frac{8}{3}.$ | 25. $\frac{4}{3} \div 5.$ |
| 6. $\frac{5}{8} \div \frac{5}{7}.$ | 16. $3\frac{4}{7} \div 8.$ | 26. $\frac{1\frac{3}{4}}{9} \div 6\frac{1}{2}.$ |
| 7. $3\frac{3}{4} \div 4\frac{1}{2}.$ | 17. $10 \div \frac{6}{7}.$ | 27. $\frac{2}{3}$ of $\frac{3}{5} \div \frac{4}{7}.$ |
| 8. $\frac{3\frac{1}{2}}{4} \div 9.$ | 18. $6\frac{1}{2} \div 2\frac{1}{6}.$ | 28. $\frac{4}{3}$ of $2\frac{1}{4} \div \frac{6}{7}.$ |
| 9. $21 \div \frac{3}{2}.$ | 19. $\frac{1\frac{1}{9}}{2} \div \frac{5}{6}.$ | 29. $\frac{8}{9} \div \frac{3}{4}$ of $\frac{3}{7}.$ |
| 10. $\frac{7}{4} \div \frac{4}{3}.$ | 20. $17\frac{3}{5} \div 11.$ | 30. $3\frac{3}{4} \div \frac{1}{3}$ of $2\frac{4}{5}.$ |
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LESSON XXIII.**Problems.**

- If a boy reads $5\frac{1}{4}$ pages of a book in an hour, in how many hours will he read $12\frac{3}{4}$ pages?
- Mr. R. bought a bag of corn containing $1\frac{7}{8}$ bushels for \$1.05. At what rate was that per bushel?
- A man sold 8 yards of cloth for $\$4\frac{1}{2}$. How much was that per yard?
- If 4 apples cost 5 cents, how many apples can be bought for 60 cents?
- How many books, at $\$3\frac{1}{2}$ per volume, can be purchased for $\$31\frac{1}{2}$?
- How many times may $2\frac{1}{2}$ gallons be drawn from a barrel that holds $31\frac{1}{2}$ gallons?

7. If $\frac{1}{5}$ of a ton of hay is worth $10\frac{1}{2}$ dollars, what is the value of 1 ton?
8. How many bushels of apples can be bought for \$44 at $\$1\frac{3}{8}$ a bushel?
9. When a train runs 150 miles in $3\frac{3}{4}$ hours, what is the rate per hour?
10. If a laborer earns $\$1\frac{1}{2}$ per day and saves $\frac{2}{3}$ of it, how long will it take him to save \$45?
11. If 1 man can do a piece of work in $10\frac{1}{2}$ days, how long will it take 8 men to do it?
12. A certain number taken $4\frac{1}{4}$ times equals $8\frac{1}{2}$. What is the number?
13. If $\frac{2}{3}$ of 14 yards of silk cost \$12, how much will $3\frac{3}{8}$ yards cost?
14. A man bought a suit of clothes for $\$29\frac{2}{5}$, which was $4\frac{1}{2}$ times as much as he had left. How much had he at first?
15. A pianist, after playing $\frac{1}{3}$ of the notes on a certain page of his music, left 120 notes unplayed. Find the number of notes on the page.
16. A wheel turns $5\frac{1}{2}$ times in going 63 feet. Find the circumference of the wheel.
17. Find the cost of a carriage when the selling price, \$198, is $1\frac{1}{2}$ times the cost.
18. When a trader sells $2\frac{3}{4}$ pounds of butter for what 4 pounds cost him, and one pound for 40 cents, what did the butter cost him per pound?
19. How many tons of coal at $\$5\frac{1}{2}$ per ton can I get for 10 sheep worth $\$5\frac{1}{2}$ apiece.
20. If 42 acres of land cost \$2100, what will $5\frac{1}{2}$ acres cost at the same rate?

21. At 20 cents a dozen, what will 15 eggs cost?
 22. How many barrels of flour, at \$6 $\frac{1}{2}$ a barrel, must be given in exchange for 50 barrels of apples worth \$6 a barrel?
 23. If 4 men do a piece of work in 12 days, how many men will do it in 1 day? In 8 days?
 24. If a boy runs 15 $\frac{1}{2}$ miles in 3 $\frac{1}{2}$ hours, how far does he run in an hour?
 25. How many barrels of potatoes, at \$1 $\frac{1}{2}$ a barrel, can I buy for \$8 $\frac{1}{2}$.
 26. If you walk at the rate of 13 miles in 3 $\frac{1}{4}$ hours, how far can you walk, at the same rate, in 13 hours?
-

LESSON XXIV.

1. A lady shopping spent $\frac{4}{9}$ of her money and had \$45 left. How much money had she at first?

Solution.—1. The equations are $\frac{5}{9} - \frac{4}{9} = \frac{1}{9}$; $\frac{1}{9} = \$45$; $\frac{4}{9} = \$9$, $\frac{5}{9} = \$81$.

2. After spending $\frac{4}{9}$ of her money, the lady had $\frac{5}{9} - \frac{4}{9}$, or $\frac{1}{9}$ of her money remaining, and this, by the condition of the problem = \$45. If $\frac{1}{9} = \$45$, $\frac{4}{9} = \frac{4}{9}$ of \$45, or \$9, and $\frac{5}{9}$, or the whole of her money = 9 times \$9, or \$81.

3. Therefore, the lady had \$81 at first.

2. I owned $\frac{4}{7}$ of a yacht, and sold $\frac{2}{7}$ of my share for \$10,000. What was the value of the yacht?

3. A man sold a horse, and received in part payment \$48.00. There remained due him $\frac{2}{7}$ of the selling price. What was the selling price of the horse?

4. If $\frac{4}{11}$ of a week's wages amount to \$5 $\frac{1}{2}$, what is the amount of a week's wages?

5. Milk is worth $\frac{2}{3}$ as much as cream. When milk is selling at 8 cents a quart, what is the price of cream?
6. Two-fifths of Albert's marbles equal $\frac{5}{6}$ of Archer's. If Archer has 48, how many has Albert?
7. What is the number, if $\frac{4}{7}$ of $\frac{7}{8}$ of it equals 26?
8. The difference between $\frac{6}{7}$ and $\frac{5}{8}$ of a number is 52. What is the number?
9. For $3\frac{1}{2}$ days' labor a man receives \$1.50 more than for $2\frac{3}{4}$ days' labor. How much does he receive per day?
10. An acre of land was sold for \$108, which was $1\frac{1}{5}$ times its cost. Find the cost?
11. A square rod contains $30\frac{1}{2}$ square yards. If $\frac{5}{11}$ of it cost \$55, how much did a square yard cost?
12. If a messenger travels 10 miles in $2\frac{1}{2}$ hours, what is his rate per hour?
13. If a piano was sold for $\frac{2}{3}$ of its cost, what was the cost of the piano, if the selling price was \$352.
14. If $\frac{2}{3}$ of a ton of coal cost $\$3\frac{3}{5}$, how much must be paid for 6 tons?
15. If 14 pounds of beef cost $\$24\frac{1}{2}$, how many pounds can be bought for \$14?
16. A furnace consumed $5\frac{1}{2}$ tons of coal in 11 weeks. How much was that per day?
17. How many strips of carpet $4\frac{1}{4}$ yards long can be cut from a roll of carpet containing 64 yards?
18. Two bicyclists are 63 miles apart. The more speedy rider pursues the other at 12 miles an hour, which is $1\frac{1}{2}$ times the speed of the other. In what time will the foremost rider be overtaken?
19. At $6\frac{1}{2}$ cents a pound, how many pounds of sugar can be bought for \$2.60?

LESSON XXV.

1. If 5 men can build a wall in $9\frac{1}{2}$ days, in how many days can 13 men build it?
2. After walking $\frac{2}{5}$ of a certain distance, and $\frac{3}{4}$ of the remainder, I had 3 miles left to walk. Find the certain distance?
3. A pole is standing 16 feet in mud and water. $\frac{2}{3}$ of the part in the water equals the part in the mud. How long is each part?
4. If \$12 is paid for 5 yards of cloth, how much can be bought for \$5.
5. When 3 quarts of milk cost as much as 4 loaves of bread, find the cost of milk per quart if bread is worth 6 cents per loaf.
6. If by adding $\frac{1}{2}$ of Sarah's age to $\frac{1}{3}$ of her age, the sum is 44 years, how old is Sarah?
7. Having a certain distance to travel I passed over it at the rate of $5\frac{1}{2}$ miles an hour. After I had travelled 6 hours I found that I had still $\frac{1}{4}$ of the distance to go. How far did I set out to travel?
8. I paid as much for 2 bushels of wheat as for 5 bushels of oats. Two bushels of oats cost me 60 cents. Find the cost of the wheat.
9. Sugar worth 5 cents a pound costs $\$9\frac{1}{2}$ a barrel. How much is sugar worth per pound that cost $\$16\frac{1}{2}$ per barrel?
10. How much corn will 9 acres produce if $4\frac{1}{2}$ acres produce 200 bushels?
11. A train of cars speeds at the rate of 100 miles in $2\frac{1}{2}$ hours. How long will it be in running 1000 miles?

12. If a furnace consumes $\frac{1}{4}$ of a ton of coal in $\frac{2}{3}$ of a month, how long will $3\frac{1}{2}$ tons last the furnace?

13. A builder expended $\frac{3}{5}$ of his money on masonry and $\frac{2}{3}$ as much for lumber. He then had left \$220. How much money had he at first?

14. A man bought a colt, and the next day a horse for $1\frac{3}{4}$ times as much. He afterwards sold both for $1\frac{1}{2}$ times their cost, \$330. How much did each cost him?

LESSON XXVI.

1. Hats that cost $\$2\frac{1}{4}$ apiece were sold for $1\frac{1}{2}$ times their cost. How many must be sold to gain $\$5\frac{3}{4}$.

2. A boy earns $\$3\frac{3}{4}$ a day and his father $1\frac{5}{8}$ times as much. How long will it take both to earn \$76.

3. If a family of 4 persons expends $\$3\frac{1}{2}$ for food daily, how long will the same sum of money last 3 persons?

4. A bicycler runs down hill at the rate of 21 miles an hour. He returns upon the same road at $\frac{5}{7}$ of that rate. How far can he go and return on such conditions in 10 hours?

5. If $4\frac{1}{2}$ cords of wood cost $\$25\frac{1}{4}$, how much will 6 cords cost?

6. A man sold a piece of cloth for \$42, by which bargain he lost $\frac{2}{5}$ of what the cloth cost him. How much did it cost him, and how much did he lose?

7. If, when the days are $9\frac{1}{2}$ hours long, a man performs a journey in 10 days, in how many days would he perform it when the days are 12 hours long?

8. If \$64 is $\frac{1}{4}$ of what was paid for wood at \$8 per cord, how many cords were bought?
9. A man expends $\frac{2}{3}$ of his money and then earns $\frac{1}{2}$ as much as he has left; he then has \$50. How much had he at first?
10. Find how far apart two persons will be in 7 hours, if they go in the same direction, one at the rate of $17\frac{1}{2}$ miles in $3\frac{1}{2}$ hours and the other at the rate of $22\frac{1}{2}$ miles in $5\frac{1}{2}$ hours.
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LESSON XXVII.

Fractional Relation.

1. 12 is $\frac{3}{4}$ of what number?

Solution.—1. The equations are $\frac{3}{4} N. = 12$; $\frac{1}{4} N. = 4$; $\frac{4}{4} N. = 16$.

2. Since $\frac{3}{4}$ of the number equals 12, $\frac{1}{4}$ of the number equals $\frac{1}{3}$ of 12, or 4; and since $\frac{1}{4}$ of the number equals 4, $\frac{4}{4}$ of the number, or the number, equals 4 times 4, or 16.

3. Therefore, 12 is $\frac{3}{4}$ of 16.

2. In like manner find of what number:

- | | | |
|---------------------------|----------------------------|-----------------------------|
| 1. 20 is $\frac{2}{3}$. | 11. 18 is $\frac{9}{11}$. | 21. 72 is $\frac{4}{3}$. |
| 2. 25 is $\frac{5}{6}$. | 12. 50 is $\frac{10}{3}$. | 22. 144 is $\frac{12}{3}$. |
| 3. 45 is $\frac{5}{9}$. | 13. 36 is $\frac{18}{5}$. | 23. 35 is $\frac{7}{2}$. |
| 4. 49 is $\frac{7}{8}$. | 14. 27 is $\frac{9}{11}$. | 24. 49 is $\frac{7}{16}$. |
| 5. 100 is $\frac{5}{7}$. | 15. 90 is $\frac{45}{7}$. | 25. 28 is $\frac{7}{20}$. |
| 6. 75 is $\frac{3}{4}$. | 16. 60 is $\frac{15}{6}$. | 26. 42 is $\frac{7}{6}$. |
| 7. 72 is $\frac{8}{5}$. | 17. 48 is $\frac{12}{5}$. | 27. 80 is $\frac{10}{3}$. |
| 8. 96 is $\frac{8}{9}$. | 18. 32 is $\frac{8}{3}$. | 28. 75 is $\frac{5}{3}$. |
| 9. 30 is $\frac{10}{6}$. | 19. 16 is $\frac{1}{2}$. | 29. 45 is $\frac{9}{7}$. |
| 10. 24 is $\frac{3}{4}$. | 20. 120 is $\frac{4}{3}$. | 30. 96 is $\frac{12}{5}$. |

3. 17 is $\frac{3}{5}$ of what number?

Solution.—The equations are $\frac{3}{5} N = 17$; $\frac{1}{5} N = \frac{17}{3}$; $\frac{5}{3} N = \frac{85}{3} = 28\frac{1}{3}$.

4. In like manner find of what number:

- | | | | |
|--|--------------------------|---------------------------|----------------------------|
| 1. 16 is $\frac{5}{6}$. | 5. 6 is $\frac{10}{3}$. | 9. 29 is $\frac{2}{3}$. | 13. 23 is $\frac{3}{7}$. |
| 2. 14 is $\frac{3}{5}$. | 6. 13 is $\frac{5}{8}$. | 10. 40 is $\frac{3}{5}$. | 14. 100 is $\frac{3}{5}$. |
| 3. 12 is $\frac{5}{9}$. | 7. 19 is $\frac{3}{4}$. | 11. 22 is $\frac{5}{8}$. | 15. 14 is $\frac{5}{20}$. |
| 4. 9 is $\frac{5}{6}$. | 8. 7 is $\frac{2}{3}$. | 12. 20 is $\frac{5}{7}$. | 16. 17 is $\frac{5}{9}$. |
| 5. 25 is $\frac{5}{6}$ of how many times 10? | | | |

Solution.—1. The equations are $25 = \frac{5}{6}$ of 40; $40 = 4$ times 10.

2. Therefore, 25 is $\frac{5}{6}$ of 4 times 10.

6. 24 is $\frac{5}{6}$ of how many times 8? 4? 16? 32? 64?
 30 is $\frac{5}{6}$ of how many times 3? 2? 6? 9? 4? 12?
 8 is $\frac{5}{6}$ of how many times $\frac{1}{3}$? $\frac{2}{3}$? $\frac{5}{6}$? $\frac{8}{9}$? $\frac{9}{10}$? $\frac{11}{12}$?
 9 is $\frac{5}{6}$ of how many times 2? 3? 4? 12? 24?
-

LESSON XXVIII.

Problems.

- If I paid \$7 a cord for wood, and if \$49 is $\frac{7}{11}$ of what I paid in all, how many cords did I buy?
- \$360 is $\frac{4}{5}$ of the cost of some land at \$81 per acre. How many acres are there?
- \$10 $\frac{1}{2}$ is $\frac{1}{10}$ of what was paid for horses at \$66 $\frac{1}{2}$ a head. How many horses were bought?
- A lady paid \$12 $\frac{1}{2}$ for a watch-chain, which was $\frac{2}{3}$ of what was paid for a watch. How much did the watch cost?

5. The cost of digging a well $37\frac{1}{2}$ feet deep was $\$3\frac{3}{5}$ per foot. This sum was $\frac{1}{4}$ of the money appropriated. How much had been appropriated?

6. How far had a man to travel if, after making an average of $41\frac{1}{2}$ miles per day for 4 days, he had accomplished but $\frac{3}{4}$ of his journey?

7. A farmer sold 6 tons of hay at $\$12\frac{3}{4}$ per ton. What he received for it was $\frac{9}{10}$ of the value of what he had left, at $\$11$ per ton. How many tons had he left?

8. $29\frac{1}{4}$ is $\frac{3}{8}$ of what was paid for 15 yards of silk. How much was the silk per yard?

9. A modiste made a dress for Hannah for $\$12\frac{1}{2}$. This sum was $\frac{25}{8}$ of 7 times what she paid for ribbons at $\$.25$ per yard. How many yards of ribbon did she buy?

10. $5\frac{1}{2}$ yards is $\frac{1}{3}\frac{1}{2}$ of $\frac{1}{10}$ of the number of yards in a mile. Find the number of yards in a mile.

LESSON XXIX.

1. $\frac{5}{8}$ of 24 is how many fourths of 20?

Solution.—The equations are $\frac{5}{8}$ of 24 = 15; $\frac{1}{4}$ of 20 = 5; 15 = 3 times 5. Therefore, 15 = $\frac{3}{4}$ of 20.

2. $\frac{6}{7}$ of 42 is how many sixths of 18?

3. $\frac{3}{4}$ of 48 is how many twelfths of 48?

4. $\frac{6}{7}$ of 49 is how many sevenths of 21?

5. $\frac{4}{7}$ of 42 is how many halves of 4?

6. $\frac{12}{13}$ of 39 is how many ninths of 27?

7. $\frac{9}{11}$ of $30\frac{1}{4}$ is how many eighths of 24?

8. $\frac{10}{9}$ of $20\frac{1}{4}$ is how many sevenths of 49?

9. $\frac{5}{8}$ of $6\frac{2}{3}$ is how many sixths of 72?
10. $\frac{6}{7}$ of $9\frac{1}{10}$ is how many thirds of 21?
11. $\frac{4}{5}$ of $5\frac{8}{11}$ is how many fourths of 28?
12. $\frac{5}{9}$ of $9\frac{9}{11}$ is how many ninths of 81?
13. $\frac{6}{11}$ of $13\frac{4}{5}$ is how many sixths of 12?
14. $\frac{7}{8}$ of $9\frac{2}{3}$ is how many thirds of 33?
15. $\frac{10}{3}$ of $11\frac{2}{5}$ is how many halves of 56?
16. 7 times $14\frac{2}{3}$ is how many thirds of 90?
17. $\frac{5}{11}$ of $24\frac{1}{6}$ is $\frac{4}{7}$ of what number?

Solution.—Equations: $\frac{4}{7} N. = \frac{5}{11}$ of $12\frac{1}{6}$; $\frac{4}{7} N. = 11$;
 $N. = \frac{77}{4} = 19\frac{1}{4}$. Therefore, $\frac{5}{11}$ of $24\frac{1}{6}$ is $\frac{4}{7}$ of $19\frac{1}{4}$.

18. $\frac{5}{7}$ of $7\frac{7}{8}$ is $\frac{2}{7}$ of what number?
 19. $\frac{7}{6}$ of $15\frac{3}{4}$ is $\frac{3}{4}$ of what number?
 20. $\frac{3}{8}$ of $22\frac{2}{5}$ is $\frac{3}{7}$ of what number?
 21. $\frac{11}{10}$ of $33\frac{1}{3}$ is $\frac{11}{12}$ of what number?
 22. $\frac{3}{4}$ of $8\frac{1}{3}$ is $\frac{3}{8}$ of what number?
 23. $\frac{1}{7}$ of $10\frac{2}{11}$ is $\frac{32}{9}$ of what number?
 24. $\frac{6}{5}$ of $18\frac{3}{4}$ is $\frac{2}{3}$ of what number?
 25. $\frac{8}{9}$ of $14\frac{1}{7}$ is $\frac{1}{2}$ of what number?
-

LESSON XXX.

Problems.

1. $\frac{4}{7}$ of $5\frac{1}{4}$ pounds of sugar was $\frac{3}{10}$ of the number of pounds of sugar that Margaret bought at 5 cents a pound. Find the cost.

2. $\frac{6}{5}$ of $\$7\frac{1}{2}$ was $\frac{3}{4}$ of what John paid for 4 pairs of shoes. How much did the shoes cost per pair?

3. $\frac{2}{5}$ of $22\frac{1}{2}$ miles is $\frac{8}{23}$ of the distance between Lancaster and Philadelphia. Find the distance.
4. If $2\frac{3}{4}$ times $8\frac{2}{5}$ is $\frac{7}{8}$ of the distance of A. from B., what is that distance?
5. I paid $\frac{5}{6}$ of \$26 $\frac{1}{2}$ for $5\frac{1}{2}$ cords of wood. How much did the wood cost per cord?
6. A man has a certain sum of money. If he had twice that sum, $\frac{6}{11}$ of it would be \$144. How much money has he?
7. If $\frac{3}{5}$ of $3\frac{3}{4}$ the distance from Lancaster to Harrisburg is 77 miles, what is the distance?
8. At the rate of \$12 $\frac{1}{2}$ per ton, a farmer received \$87 $\frac{1}{2}$ for $\frac{7}{12}$ of his hay. How many tons had he?
9. At \$60 per acre, \$520 was received for $1\frac{3}{8}$ of some land. How many acres of land were there?
10. A horse was valued at \$250. $\frac{3}{50}$ of this sum was $\frac{3}{5}$ of the money lost on him when he was sold. How much was received for him?
11. A man paid \$600 for a piano. $\frac{3}{5}$ of $2\frac{1}{2}$ times this sum is 5 times the money he paid for music lessons. How much did the lessons cost?
12. The moon is distant from the earth about 240,000 miles. $\frac{3}{40}$ of this distance is $2\frac{1}{4}$ times the diameter of the earth. What is the diameter of the earth?
13. A field contains 100 trees and a certain number of cattle. $\frac{3}{5}$ of the number of trees equals $\frac{5}{4}$ of the number of cattle. What are the cattle worth at \$20 a head?
14. A year contains 365 days, and $\frac{2}{3}$ of this number is $\frac{1}{46}$ of 73 times the number of days in summer. How many days in summer?
15. $\frac{4}{5}$ of my age is $\frac{8}{11}$ of 33 years. How old am I?

16. If $\frac{3}{4}$ of an acre of land cost $\frac{2}{3}$ of \$180, what will $\frac{3}{4}$ of an acre cost?

17. How much money had a laborer at first if, after expending $\frac{3}{5}$ of his money and earning $\frac{4}{5}$ as much as he expended, he had \$42?

18. The books in a gentleman's library are worth \$3000. $\frac{5}{6}$ of this sum is $\frac{5}{3}$ of $\frac{1}{2}$ the value of his horse. What is the value of his horse?

19. A furniture dealer sold desks to the amount of \$270. $\frac{5}{6}$ of this sum is $\frac{5}{7}$ of what the desks cost. Find the cost.

20. If by adding $\frac{6}{11}$ of my age to $\frac{3}{11}$ of $\frac{3}{5}$ of it, the sum will be 39 years, what is my age?

21. A boy studying a school history has read $\frac{2}{3}$ of $\frac{1}{2}$ of it, but lacks 160 pages of the end. How many pages in the book?

22. A carriage costs \$250 and $\frac{2}{3}$ of $\frac{5}{6}$ of this is $\frac{5}{2}$ of what two horses cost. Find the cost of the horses.

23. A number increased by 51 more than $\frac{5}{6}$ of it equals 172. What is the number?

24. If $\frac{2}{3}$ of $4\frac{1}{2}$ pounds of rice cost $\frac{1}{2}$ of $\frac{2}{3}$ of 60 cents, what is the cost of the rice per pound?

25. If $3\frac{3}{4}$ cords of wood cost $\frac{5}{6}$ of \$18, how much is the cost per cord?

LESSON XXXI.

1. What part of 7 is 3?

Solution.—1. $1 = \frac{1}{1}$ of 7; $3 = \frac{3}{1}$ of 7.

2. Since 1 is $\frac{1}{7}$ of 7, 3 is 3 times $\frac{1}{7}$ of 7, or $\frac{3}{7}$ of 7.

2. What part of 6 is 5? Of 6 is 4? Of 8 is 5?
Of 9 is 7?

3. What part of 10 is 7? Of 9 is 4? Of 9 is 6?
Of 15 is 10?

4. Find what part:

- | | | |
|--------------|--------------|--------------|
| 9 is of 12. | 6 is of 18. | 16 is of 24. |
| 10 is of 15. | 10 is of 18. | 18 is of 24. |
| 10 is of 16. | 14 is of 18. | 21 is of 28. |
| 8 is of 18. | 15 is of 20. | 25 is of 35. |
| 12 is of 16. | 15 is of 25. | 24 is of 30. |

5. What part of:

- | | | |
|-----------|------------|-----------|
| 6 is 4? | 60 is 40? | 14 is 6? |
| 36 is 24? | 72 is 48? | 16 is 10? |
| 45 is 30? | 90 is 60? | 20 is 12? |
| 50 is 40? | 100 is 80? | 36 is 28? |
| 60 is 45? | 120 is 90? | 56 is 42? |

6. What part of:

- | | | |
|----------------------------------|-------------------------------------|---------------------------------------|
| $\frac{4}{5}$ is $\frac{1}{5}$? | $\frac{6}{9}$ is $\frac{3}{9}$? | $\frac{24}{15}$ is $\frac{6}{15}$? |
| $\frac{6}{7}$ is $\frac{2}{7}$? | $\frac{9}{11}$ is $\frac{3}{11}$? | $\frac{86}{80}$ is $\frac{9}{80}$? |
| $\frac{6}{7}$ is $\frac{3}{7}$? | $\frac{10}{14}$ is $\frac{5}{14}$? | $\frac{51}{100}$ is $\frac{3}{100}$? |

7. What part of:

- | | | |
|------------------------------------|------------------------------------|------------------------------------|
| $3\frac{1}{2}$ is $\frac{3}{2}$? | $5\frac{1}{4}$ is $2\frac{1}{4}$? | $4\frac{3}{4}$ is $2\frac{1}{4}$? |
| $4\frac{1}{2}$ is $1\frac{1}{2}$? | $3\frac{3}{5}$ is $2\frac{1}{5}$? | $3\frac{1}{3}$ is $2\frac{1}{3}$? |
| $4\frac{2}{5}$ is $2\frac{2}{5}$? | $4\frac{4}{5}$ is $2\frac{2}{5}$? | $7\frac{1}{2}$ is $2\frac{1}{2}$? |

8. What part of $\frac{4}{5}$ is $\frac{3}{4}$?

Solution.—Equations: $\frac{4}{5} = \frac{16}{20}$; $\frac{3}{4} = \frac{15}{20}$; $\frac{15}{20} = \frac{15}{16}$ of $\frac{16}{20}$.

What part of:

- | | | |
|----------------------------------|----------------------------------|------------------------------------|
| $\frac{4}{5}$ is $\frac{1}{2}$? | $\frac{4}{5}$ is $\frac{2}{3}$? | $2\frac{3}{5}$ is $1\frac{1}{2}$? |
| $\frac{3}{4}$ is $\frac{2}{3}$? | $\frac{4}{5}$ is $\frac{1}{2}$? | $3\frac{1}{2}$ is $1\frac{3}{4}$? |
| $\frac{3}{4}$ is $\frac{2}{5}$? | $\frac{4}{5}$ is $\frac{2}{3}$? | $4\frac{4}{5}$ is $3\frac{1}{2}$? |

LESSON XXXII.

1. If 9 boys earn \$57 in a week, how much will 6 boys earn in the same time?

Solution.—1. Equations: $6 = \frac{2}{3}$ of 9; $\frac{2}{3}$ of \$57 = \$38.

2. Since 9 boys earn \$57, 6 boys, or $\frac{2}{3}$ of 9 boys, will earn $\frac{2}{3}$ of \$57, or \$38.

2. How much will 28 pounds of sugar cost if 7 pounds cost 40 cents?

3. If 10 cords of wood cost \$55, how much will 4 cords cost?

4. If 9 loads of ashes cost \$6.00, what will 15 loads cost?

5. If a boy weighs 65 pounds and his father weighs 130 pounds, the boy's weight is what part of his father's weight?

6. The boy receives $\$1\frac{1}{2}$ per day and the father $\$2\frac{1}{2}$ per day. When the father has earned \$21, how much has the boy earned, both working daily?

7. If $\frac{3}{4}$ of a dozen of oranges cost 16 cents, how much will $3\frac{1}{2}$ dozen cost?

8. If $4\frac{1}{2}$ acres of land cost \$500, how much do $7\frac{1}{2}$ acres cost?

9. If $2\frac{1}{4}$ acres of land yield $4\frac{1}{2}$ tons of hay, how much do $5\frac{1}{4}$ acres yield?

10. $\frac{1}{6}$ of $5\frac{5}{8}$ bushels of oats was $\frac{3}{4}$ of the number of oats a farmer sold at 40 cents a bushel. What did he receive for the oats sold?

11. If a tree is $66\frac{2}{3}$ feet high and a house near it is $\frac{2}{3}$ as high, how high is the house?

12. If a boy spent in study $2\frac{2}{3}$ hours on Monday and $\frac{2}{3}$ as long on Tuesday, how long did he study on Tuesday?
13. If a man travels $14\frac{2}{5}$ miles in $4\frac{1}{2}$ hours, how far can he travel in $6\frac{1}{2}$ hours?
14. If a ton of hay can be bought for $\$12\frac{2}{3}$, what part of a ton can be bought for $\$6\frac{1}{3}$?
15. If 5 men can do a piece of work in 5 days, in how many days can 4 men do the work?
16. By working 10 hours a day a man can do a piece of work in 24 days. How long will it take him to do the work by working 8 hours per day?
17. If $5\frac{1}{2}$ tons of hay last 6 horses a month, how long will 11 tons last 11 horses?
18. If a man and his wife consume a barrel of flour in $13\frac{1}{2}$ weeks, in what time will 3 men and their wives consume a barrel of flour?
19. If 2 horses eat $3\frac{1}{2}$ bushels of oats in a week, in what time will 8 horses eat 48 bushels?
20. When $12\frac{2}{3}$ yards of silk, at $\$1.80$ per yard, are exchanged for $25\frac{1}{2}$ bushels of wheat, what is the value of the wheat per bushel?
21. When Mary has earned $\$8$, Margaret has earned $\$6$. When both have earned $\$80$, how much has each earned?
22. A. and B. are advancing towards each other, A. with the speed of 6 miles per hour and B. with the speed of 9 miles per hour. At meeting it is found that one has traveled 60 miles farther than the other. How far apart were they at starting?
23. If 5 horses can eat $26\frac{1}{2}$ loads of hay in a year, what will 8 horses eat in the same time?

24. If a horse consumes $3\frac{1}{4}$ bushels of oats in 2 days, how much will 2 horses consume in 5 days?

25. If a staff 5 feet long casts a shadow 4 feet long at 4 P.M., what is the length of a pole that casts a shadow 67 feet long at the same time?

LESSON XXXIII.

1. A. can do a piece of work in 6 days and B. can do it in 5 days. In what time can they both do it?

Solution.—1. Equations: $\frac{1}{6} + \frac{1}{5} = \frac{5+6}{30} = \frac{11}{30}$; $\frac{11}{30}$ in 1 day = $\frac{30}{11}$ in $2\frac{8}{11}$ days.

2. Since A. can do the work in 6 days, in 1 day he can do $\frac{1}{6}$ of it. Since B. can do the work in 5 days, in 1 day he can do $\frac{1}{5}$ of it. Both can do in 1 day $\frac{1}{6} + \frac{1}{5} = \frac{11}{30}$. Since in 1 day both can do $\frac{11}{30}$, they can do $\frac{30}{11}$ in as many days as 11 is contained times in 30, or $2\frac{8}{11}$.

2. C. and D. can do a piece of work in 5 days, and C. can do it alone in 7 days. How long will it take D. to do the work?

3. E. can cut a cord of wood in $\frac{2}{3}$ of a day and F. can cut a cord of wood in $\frac{3}{2}$ of a day. In what time can they both cut a cord?

4. If a bicycler can ride 32 miles in $3\frac{1}{2}$ hours, how far, at that rate, can he ride in $9\frac{1}{2}$ hours?

5. In a certain church choir $\frac{2}{3}$ of $\frac{5}{6}$ of the members are ladies. How many gentlemen are in the choir, if there are in all 32 members?

6. If a laborer does a piece of work in 18 days for \$70, how much should he receive for $7\frac{1}{2}$ days' work?

7. If $\frac{1}{7}$ of M feet of lumber cost \$6 $\frac{2}{3}$, what is the value of $\frac{3}{7}$ of M feet? [M = 1000.]

8. Three-fourths of a farm was sold at the rate of \$50 per acre, and the receipt from the sale was \$2100. How many acres did the farm contain?

9. G. can earn \$20 in $4\frac{2}{3}$ days and H. in 7 days. In how many days can they together earn \$100?

10. Two boys can do a piece of work in 5 days and 8 days respectively. In what time can they both do it?

11. A. can do a piece of work in 3 days, B. can do it in 4 days, and C. can do it in 5 days. In how many days can all together do the work?

LESSON XXXIV.

1. A father and son undertake to do a piece of work. If, on its completion, they receive \$48, how much did each earn, assuming that the father could do the work alone in $3\frac{1}{2}$ days and the son in $4\frac{1}{2}$ days?

Solution.—Equations: $3\frac{1}{2} = \frac{7}{2}$; $4\frac{1}{2} = \frac{9}{2}$. Hence, the father did $\frac{2}{7}$ in 1 day, the son $\frac{2}{9}$, and both $\frac{2}{7} + \frac{2}{9} = \frac{18+14}{63} = \frac{32}{63}$. Therefore, the father earned $\frac{18}{63}$ or $\frac{1}{6}$ of \$48 = \$27, and the son $\frac{14}{63}$ or $\frac{7}{16}$ of \$48 = \$21.

2. A. can build a wall in $2\frac{1}{2}$ days and B. can build it in $3\frac{1}{2}$ days. Assuming that they build the wall together and receive therefore \$23, how much should each receive?

3. Two men contract to labor together and at the end received \$121. How was this sum justly divided between

them if one did in $3\frac{1}{2}$ days as much as the other in $4\frac{1}{2}$ days?

4. If 6 men can dig a trench in $4\frac{1}{2}$ days, in what time can 4 men dig a similar trench?

5. A. and B. are collectors. As often as A. collects \$3 B. collects \$4. How much does each collect when both collect \$147?

6. What number is that which being increased by $\frac{5}{11}$ of itself and 37 more equals 453?

7. Paul had $2\frac{1}{2}$ times as much money as Richard. He spent $\frac{1}{4}$ of what he had, earned \$10, and then both had \$70. How much had each?

8. I sold $\frac{3}{4}$ of my ducks and $\frac{1}{2}$ of the remainder. If 3 were left, how many had I at first?

9. If 4 men do a piece of work in 12 days, how long will it take 3 men to do a piece of work 3 times as great?

10. Two employes earn respectively \$4 and \$3 per day. At the end of a certain period they receive \$210. How much of this sum did each earn?

11. A piece of work cost in all \$280. When $\frac{3}{4}$ of it was completed three men contracted to finish it and worked 1, 2, and 4 days, respectively. How much should each receive?

12. An apple-vender sold his fruit 4 for 5, but bought it 6 for 5. When he had gained a dollar, how much fruit had he sold?

13. A woodman felled $\frac{5}{13}$ of his trees and had left standing 135 more trees than he felled. How many trees had he?

14. A trolley-car runs 20 miles in $1\frac{1}{2}$ hours. At that rate how far will it go in $5\frac{1}{2}$ hours?

LESSON XXXV.

1. The sum of the ages of A., B., and C. is 147 years. What is the age of each, if A. is $\frac{3}{5}$ as old as B. and B. is $\frac{3}{5}$ as old as C.?

2. How much can A. and B. together mow in 7 days, if A. can mow 10 acres in $\frac{1}{2}$ a day and B. 12 acres in $\frac{3}{4}$ of a day?

3. A grocer exchanges lard for butter, giving 5 pounds of lard for 2 pounds of butter. Assuming that lard is worth 10 cents a pound, what are $2\frac{1}{2}$ pounds of butter worth?

4. Three legatees, A., B., and C., received \$1480, as follows: A. $\frac{1}{4}$ as much as B., and B. $\frac{1}{4}$ as much as C. How much did each receive?

5. After selling $\frac{1}{3}$ of my fowls, and then $\frac{1}{2}$ of the remainder, I had 45 fowls left. How many had I at first?

6. A pedestrian travels 25 miles in $8\frac{1}{2}$ hours. How far at that rate can he travel in $10\frac{1}{4}$ hours?

7. Matthew is 32 years older than Richard. How old is each if Richard is $\frac{2}{3}$ as old as Matthew?

8. Mr. Henry has a monthly income. After expending \$90 per month and earning $\frac{1}{2}$ of what is left, he has \$75. Find his monthly income.

9. $\frac{2}{3}$ of the number of acres in a field is 17 acres less than $\frac{1}{3}$ of the number of acres in a field twice as large. How many acres in the field?

10. Two-thirds of a number added to $\frac{3}{5}$ of the same number, and the sum multiplied by $2\frac{1}{2}$, yields a product of 95. Find the number.

11. A man owning $\frac{1}{4}$ of a mine sold $\frac{2}{3}$ of his share for \$21,000. From this sale estimate the value of the mine.

12. A man gave a gift of money to his 3 sons. To the oldest, $\frac{1}{2}$ of the money; to the second, $\frac{1}{3}$ of it; and to the third, \$30 less than to the second. How much did each receive ?

LESSON XXXVI.

1. When eggs are bought at 30 cents a dozen, and sold again so as to gain 1 cent on every 5 cents, how many are sold for 27 cents ?

2. Find how much beef, at $16\frac{2}{3}$ cents a pound, can be bought for \$5 ?

3. When silk is worth \$2.50 for $1\frac{1}{4}$ yards, find how much can be bought for \$41.

4. A landau was bought for \$240, which was $\frac{1}{5}$ less than its value, and sold again for $\frac{1}{3}$ more than its value. What was the gain ?

5. Three drovers, A., B., and C., had cattle in a pasture as follows: A., $\frac{1}{2}$ of the number; B., $\frac{1}{3}$; and C., 24 less than both A. and B. What was the number of cattle in the pasture ?

6. How much of \$50 was left, after paying John for 16 days' work, at $\$1\frac{1}{2}$ per day, and Henry for 15 days' work, at $\$1\frac{1}{8}$ per day ?

7. A horse cost \$150, which was $\frac{4}{5}$ of 6 times the cost of the harness. What was the cost of the harness ?

8. A miller sold 16 barrels of flour for \$108, and gained $\$2\frac{1}{2}$ on each barrel. How much was the cost per barrel ?

9. One dollar's worth of ribbon contained $3\frac{3}{4}$ yards bought at one time and $2\frac{1}{2}$ yards bought at another time at the same price. What was the cost per yard?
10. If $4\frac{1}{2}$ yards of cloth cost \$26, how much will $18\frac{3}{4}$ yards cost?
11. Of a quantity of hay, five tons were sold for \$76, and one-half the remainder at $\$14\frac{1}{2}$ per ton. If \$208 was received for all, how many tons were sold?
12. Coal was exchanged for wood at $\$3\frac{3}{5}$ per cord. The wood delivered equals $2\frac{2}{3}$ times the value of a ton of coal. If there were $3\frac{3}{5}$ cords of wood, what was the value of a ton of coal?
13. $2\frac{3}{4}$ times $\$9\frac{1}{2}$ is $\frac{3}{5}$ of the sum A. owes B. How much does A. owe B.?

LESSON XXXVII.

1. Two colts cost a farmer \$80. If one cost $\frac{2}{3}$ as much as the other, what did each cost?
2. $\frac{4}{5}$ of a number plus $\frac{1}{2}$ of $\frac{2}{3}$ of 24 is 24. Find the number.
3. Hannah is 25 years old and $\frac{4}{5}$ of her age is $\frac{1}{2}$ of $3\frac{3}{5}$ times Sarah's age. How old is Sarah?
4. If you divide $\frac{4}{5}$ of a certain number by $2\frac{1}{2}$, the quotient will be 51. Find the number.
5. A clock was sold at a gain of $\frac{1}{4}$ of its cost. If \$6 more had been gained, the whole gain would have been $\frac{1}{3}$ of the cost. Find the cost.
6. If an automobile carriage can run 90 miles in $4\frac{1}{2}$ hours, how long at that rate will it take to run 240 miles?

7. A hogshead has a pipe that fills it in $\frac{1}{4}$ of an hour, and another that empties it in 1 hour. If the hogshead is empty and both pipes are allowed to run, in what time will it be filled?

8. A man spent in travel $\frac{3}{5}$ of his money, and then collected a bill equal in amount to $\frac{1}{4}$ of what he had left, but lacked \$150 of having the sum he started with. How much money did he spend in travel?

9. A farmer sold 6 barrels of potatoes for $\$22\frac{1}{2}$, a sum equal to $\frac{5}{6}$ of what he received for the remainder at $\$4\frac{1}{2}$ per barrel. How many barrels of potatoes did he sell?

10. Two students solved problems, but with unequal results, one having to his credit $\frac{3}{5}$ of the whole number, while the other had 20 less, which was $\frac{1}{2}$ of the remainder. How many problems did each solve?

LESSON XXXVIII.

1. When a ten-foot pole casts a shadow $16\frac{1}{2}$ feet long, what is the length of the shadow of a tree $33\frac{1}{2}$ feet high?

2. $\frac{4}{11}$ of the people in a village are women, $\frac{3}{11}$ are men, and the rest are children. If the children number 140, how many men and women are there?

3. A piece of work can be done by 10 men in $7\frac{1}{2}$ days. In how many days can the work be done by 7 men?

4. I paid \$290 for a watch and chain, but the watch cost $6\frac{1}{4}$ times as much as the chain. How much did each cost?

5. A man after losing $\frac{2}{5}$ of his money perceived that

\$12 was $\frac{3}{4}$ of what remained. How much money had he before losing?

6. A boy found $\frac{3}{7}$ as many marbles as he already had. He then gave $\frac{1}{2}$ of his number to his brother and had 70 left. How many had he at first?

7. The dial of a clock has 12 equal spaces. When the minute hand passes over 12 spaces the hour hand passes over but one space. How many spaces, therefore, may the minute hand be considered as having gained in one hour, or 60 minutes?

8. If the minute hand gains 11 spaces in 60 minutes, it will gain one space in how many minutes?

9. The two hands are two spaces apart at 2 o'clock. In how many minutes will they be together?

10. Two foot-travellers are 50 steps apart, their steps being equal. While one takes 3 steps the other follows with 5. How many steps will the one in advance go before he is overtaken?

11. If a mule eats $\frac{4}{5}$ of a ton of hay in a month, compute the hay bill for 7 mules for $3\frac{1}{2}$ months at \$9 per ton.

12. If $\frac{5}{6}$ of a yard of silk costs \$2 $\frac{5}{6}$, how much can be bought for \$23?

13. \$100 is paid for $11\frac{2}{3}$ weeks' work. How much should be paid for $17\frac{1}{4}$ weeks' work?

14. In how many hours will two trains be together if they are moving in the same direction, are 34 miles apart, and the faster train gains $1\frac{2}{3}$ miles per hour?

15. Of a ship's company, $\frac{3}{4}$ are on duty, $\frac{1}{10}$ are sick, $\frac{3}{5}$ of the remainder are on leave. If, in addition to these, 6 are missing, what is the number of men on the books?

LESSON XXXIX.

1. \$5 is paid for sawing sticks of wood into 2 pieces. How much should be paid for sawing each stick into 3 pieces?

2. When flour at $\$6\frac{2}{3}$ per barrel furnishes 6-cent loaves, what will be the price of bread per loaf, when flour is $\$8\frac{1}{2}$ per barrel?

3. How many gallons of vinegar, at $16\frac{2}{3}$ cents a gallon, can be bought for \$2?

4. A man walked $\frac{2}{3}$ of a mile; he then retraced his steps for 40 rods and was $\frac{11}{10}$ of a mile from his starting point. How many rods in a mile?

5. Wood is exchanged for coal, $10\frac{1}{2}$ cords of wood being reckoned equal to $7\frac{1}{2}$ tons of coal. If wood is \$5 per cord, what is the value of 20 tons of coal?

6. \$100 was received for some swine that cost \$5 per head, and their owner thus lost $\frac{1}{11}$ of his investment in them. How many swine did he sell?

7. Fowls that were bought at $\$1\frac{1}{2}$ each were sold at \$21 a dozen. How much was gained on 200 fowls?

8. How much will $13\frac{1}{2}$ pounds of beef cost if 11 pounds cost \$1.98?

9. I bought $2\frac{3}{4}$ tons of coal for $\$9\frac{1}{2}$ and sold $1\frac{1}{2}$ tons at the same rate. How much did I get for it?

10. $\frac{4}{5}$ of $\frac{3}{4}$ of a number is 25 more than $\frac{3}{2}$ of $\frac{2}{3}$ of the number. Find the number.

11. A man expended $\frac{4}{5}$ of his money and found that $\frac{2}{3}$ of what was left equaled \$90. How much had he at first?

12. How long will it take 6 men to do a piece of work if 7 men can do the work in $4\frac{2}{3}$ days?

13. If Hannah can make 2 dresses in 5 days, how many dresses can she make in $12\frac{1}{2}$ days?

14. When $5\frac{1}{2}$ tons of coal cost \$44, what do $1\frac{1}{4}$ tons cost?

15. A man sold $\frac{1}{3}$ of his pigeons, bought $1\frac{1}{2}$ times as many as he sold, and then had 42. How many had he at first?

16. A lady exchanged eggs for muslin, supplying them at the rate of 9 for 15 cents. How many eggs paid for 24 yards of muslin sold at the rate of 4 yards for half a dollar?

17. Find the cost of my horse, knowing that $\frac{1}{3}$ of its cost is $\$12\frac{1}{2}$ more than $\frac{5}{4}$ of its cost.

18. A man travelling from village A. to village D. passed through villages B. and C. The distance from A. to B. is $\frac{2}{3}$ of the distance from B. to C. and $\frac{3}{4}$ of the distance from C. to D. If the distance from B. to D. is 24 miles, what is the distance from A. to B.?

19. An author in writing a book wrote $\frac{1}{3}$ of it the first month, $\frac{2}{3}$ of the remainder the second month, and the final 100 pages the third month. How many pages did the book contain?

20. A bushel of wheat and a bushel of corn cost together $\$1\frac{1}{4}$. If 10 bushels of wheat cost $\$2\frac{1}{2}$ more than 10 bushels of corn, what is the cost of each per bushel?

21. An audience listening to a lecturer numbered 770 persons. $\frac{4}{5}$ of the number of women present equalled $\frac{2}{3}$ of the number of men. Find the number of each.

22. How much money had a man at first if, after giving $\frac{2}{3}$ of it for a house and \$540 for furniture, he had $\frac{1}{4}$ of his money remaining?

23. A fruit vender bought oranges at the rate of 15 cents a dozen, and sold them at the rate of 2 for 5 cents. How much did he make on each orange? How much did he make on a dozen?

24. The sum of a father's and a son's age is 87 years. If $\frac{7}{11}$ of the father's age equals $\frac{1}{4}$ of the son's, how old is each?

25. A lady shopping expended \$10 more than $\frac{3}{5}$ of her money and had $\$12\frac{1}{2}$ left. How much money had she at first?

LESSON XL.

Aliquot Parts.

Aliquot Parts are numbers that are contained an integral number of times in the given number.

1. What part of \$1.00 is each of the following:

- 25 cents? 50 cents? 75 cents? $12\frac{1}{2}$ cents?
- $37\frac{1}{2}$ cents? $62\frac{1}{2}$ cents? $87\frac{1}{2}$ cents? $6\frac{1}{4}$ cents?
- $18\frac{3}{4}$ cents? $(12\frac{1}{2} \text{ cents} + 6\frac{1}{4} \text{ cents})$? $33\frac{1}{8}$ cents?
- $8\frac{1}{2}$ cents? $2\frac{1}{2}$ cents? $3\frac{1}{3}$ cents? $16\frac{2}{3}$ cents?
- $14\frac{2}{3}$ cents? 20 cents? 40 cents? 60 cents?

2. What will lunches for 8 persons cost at 25 cents each?

1. The equations are: 25 cents = $\$1\frac{1}{4}$; $\$1\frac{1}{4} \times 8 = \frac{8}{4} = \2.00 .
2. Since 1 lunch costs $\$1\frac{1}{4}$, 8 lunches will cost 8 times $\$1\frac{1}{4}$, which equals $\$2\frac{1}{4}$ or \$2.
3. Therefore, 8 lunches at 25 cents each will cost \$2.00.

3. If a boy's wages are 75 cents per day, how much does he earn in 88 days?

4. How much will 18 pounds of sugar plums cost at $66\frac{2}{3}$ cents a pound?

5. How much will 64 bushels of wheat cost at $87\frac{1}{2}$ cents per bushel?

6. How much more will 6 pecks of peaches cost at $66\frac{2}{3}$ cents a peck than at $33\frac{1}{3}$ cents a peck?

7. At a book sale 75 volumes were sold for $37\frac{1}{2}$ cents each. What was received for them?

8. If a pound of tea costs $66\frac{2}{3}$ cents, what do 45 pounds cost?

9. If a boy receives for his services $3\frac{1}{2}$ cents an hour for 8 hours daily, how much will he have earned at the end of each week?

10. What part of:

100 is 25?	1000 is 250?	10 is 2.50?	100 is $12\frac{1}{2}$?
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100 is 5?	1000 is 500?	10 is .25?	100 is $37\frac{1}{2}$?
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100 is 50?	1000 is 750?	10 is 5?	100 is $62\frac{1}{2}$?
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100 is 75?	1000 is 125?	10 is 7.50?	100 is $87\frac{1}{2}$?
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100 is $33\frac{1}{3}$?	1000 is 375?	10 is $3\frac{1}{3}$?	100 is $16\frac{2}{3}$?
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100 is $66\frac{2}{3}$?	1000 is $666\frac{2}{3}$?	10 is $.8\frac{1}{3}$?	100 is $8\frac{1}{3}$?
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100 is $14\frac{2}{3}$?	1000 is 625?	10 is $6\frac{2}{3}$?	100 is $6\frac{1}{3}$?
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11. What is the value of $3\frac{1}{2}$ times 51?

Solution.—The equations are: $3\frac{1}{2} = \frac{1}{2}$ of 10; 10 times 51 = 510; $\frac{1}{2}$ of 510 = 170.

12. Find the value of:

1. $32 \times 12\frac{1}{2}$.	6. $51 \times 66\frac{2}{3}$.	11. $144 \times 6\frac{1}{4}$.
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2. $56 \times 37\frac{1}{2}$.	7. 48×25 .	12. $126 \times 8\frac{1}{3}$.
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3. 28×75 .	8. $84 \times 33\frac{1}{3}$.	13. $156 \times 16\frac{2}{3}$.
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4. $64 \times 87\frac{1}{2}$.	9. $77 \times 14\frac{2}{3}$.	14. 320×375 .
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5. $96 \times 62\frac{1}{2}$.	10. $99 \times 6\frac{2}{3}$.	15. $333 \times 666\frac{2}{3}$.
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13. Find the value of:

- | | | |
|---------------------|-----------------------|--------------------------------|
| 1. $64 \times 125.$ | 8. $86 \times 250.$ | 15. $112 \times 250.$ |
| 2. $48 \times 250.$ | 9. $44 \times 125.$ | 16. $8 \times 2.50.$ |
| 3. $88 \times 375.$ | 10. $16 \times 875.$ | 17. $16 \times .25.$ |
| 4. $80 \times 750.$ | 11. $144 \times 125.$ | 18. $76 \times 5.$ |
| 5. $96 \times 875.$ | 12. $80 \times 375.$ | 19. $12 \times 7.50.$ |
| 6. $48 \times 625.$ | 13. $96 \times 625.$ | 20. $36 \times 6\frac{2}{3}.$ |
| 7. $32 \times 375.$ | 14. $128 \times 500.$ | 21. $98 \times 14\frac{2}{3}.$ |

14. How much will one dozen plum trees cost at \$.25 apiece?

Solution.—Equations: $\$.25 = \$\frac{1}{4}$; $\frac{1}{4} \times 12 = \frac{12}{4} = \$3.00.$

- 15.** How much will 48 swine cost at \$3.75 per head?
- 16.** How much will 56 cords of wood cost at \$6.25 a cord?
- 17.** How much will 36 tons of coal cost at \$6.66 $\frac{2}{3}$ per ton?
- 18.** Find the cost of 18 pairs of shoes at \$3.33 $\frac{1}{3}$ per pair.
- 19.** Find the cost of 2 dozen hats at \$2.50 apiece.
- 20.** What is the cost of hose per dozen at \$.16 $\frac{2}{3}$ a pair?
- 21.** Twenty-five pairs of pantaloons will cost what, at \$2.50 a pair?
- 22.** Find the cost of 25 railroad tickets at \$3.75 apiece.
- 23.** If the wages for a month are \$37.50, what will the wages for 2 years amount to?
- 24.** At \$.08 $\frac{1}{3}$ apiece what will 144 pen-holders cost?
- 25.** If a mechanic earn \$3.33 $\frac{1}{3}$ per day, how much does he earn in 312 days?
- 26.** At $37\frac{1}{2}$ cents per hour, how much will a man earn in 12 days of 10 hours each?
- 27.** What cost 108 books at $11\frac{1}{2}$ cents apiece?

LESSON XLI.

1. What part of \$10 is each of the following:

1. \$1.25? [SUGGESTION.—\$1.25 is $\frac{1}{8}$ of \$10.]

2. \$3.75? 4. \$8.75? 6. \$3.33 $\frac{1}{3}$?

3. \$6.25? 5. \$1.66 $\frac{2}{3}$? 7. \$6.66 $\frac{2}{3}$?

2. What part of \$10 is each of the following:

1. \$11.25? 5. \$21.25? 9. \$28.75? 13. \$35.00?

2. \$13.75? 6. \$23.25? 10. \$27.50? 14. \$22.50?

3. \$18.75? 7. \$26.25? 11. \$32.50? 15. \$26.25?

4. \$16.25? 8. \$17.50? 12. \$33.75? 16. \$28.75?

3. If 1 barrel of flour costs \$6.25, what will 24 barrels cost?

Solution.—Equations: 1. $\$6.25 = \frac{1}{8}$ of \$10; 24 barrels at \$10 cost \$240; $\frac{1}{8}$ of \$240 = \$150.

2. Therefore, at \$6.25, 24 barrels of flour cost \$150.

4. Find the cost of 24 cows at \$23.75 a head.

SUGGESTION.—\$23.75 = $\frac{1}{4}$ of \$10.

5. If \$6.25 is paid for the use of money for a year, how much must be paid for the use of the same sum for 25 years?

6. At the cost of \$28.75 per acre, what will 32 acres of land cost?

7. At \$17.50 a week, how much will a man earn in 52 weeks?

8. If a bicycle can be bought for \$33.75, for how much can 56 bicycles of like grade be bought?

9. Clothing bought at \$21.25 per suit and sold at \$30 per suit yields how much gain on 36 suits?

10. What is $\frac{1}{2}$ of 1000? What is $\frac{2}{3}$? $\frac{3}{4}$? $\frac{5}{6}$? $\frac{7}{8}$? $\frac{9}{10}$?

11. How many 8ths of 1000 in 750? In 1125? In 1250? In 1375?

12. Find the product of 144 multiplied by 1125?

Solution.—Equations: 1. $1125 = \frac{3}{8}$ of 1000; $144 \times 1000 = 144,000$; $\frac{3}{8}$ of 144,000 = 162,000.
2. Therefore, $144 \times 1125 = 162,000$.

13. In like manner find the value of:

- | | | |
|-----------------------|-----------------------|-------------------------|
| 1. 8×375 . | 6. 32×750 . | 11. 144×1875 . |
| 2. 16×625 . | 7. 48×1375 . | 12. 328×375 . |
| 3. 24×750 . | 8. 72×1125 . | 13. 496×125 . |
| 4. 36×1125 . | 9. 88×1625 . | 14. 1288×250 . |
| 5. 56×125 . | 10. 96×375 . | 15. 1136×625 . |

14. What is the cost of 125 barrels of flour at \$6.48 a barrel?

SUGGESTION.— $\$6.48 \times 1000 = \6480 .

15. What will 375 yards of cloth cost at \$4.56?

16. If 1 house costs \$1375, what will 56 houses of like construction cost?

17. If a man earns \$1875 in a year, how much will he earn in 16 years?

18. If a barrel of flour weighs 196 pounds, what will 625 barrels weigh?

19. How much will 125 shares of railroad stock cost at \$125 per share?

20. What will a year's rental amount to at the rate of \$1125 per month?

21. Reckoning each year equal to 365 days, find the number of days in 1625 years.

22. If \$75 is paid to each soldier, how much money will be paid to 3 times 625 soldiers?

LESSON XLII.

1. How many subscribers at \$1.12 $\frac{1}{2}$ each will be required to subscribe the total sum of \$144?
2. How long must I work at the rate of \$2.37 $\frac{1}{2}$ per day to earn \$152?
3. How many tons of coal, sold at \$5.33 $\frac{1}{2}$ per ton, can be bought for \$320?
4. How many cords of wood, sold at \$3.87 $\frac{1}{2}$ per cord, can be bought for \$310?
5. How many pounds of sugar, at 4 $\frac{1}{2}$ cents a pound, can be bought for \$10?
6. If a man pays each week an installment of \$4.75, how long will it take him to pay \$190?
7. At \$31.25 per acre, find how many acres \$1000 will buy?
8. When hay costs \$12.66 $\frac{2}{3}$ per ton, what amount of hay will \$228 buy?
9. The dividend is 234,000, the quotient is 1625; find the divisor.

SUGGESTION.—1625 is $\frac{1}{7}$ of 1000; $234,000 \div 1000 = 234$.

10. If I earn \$15.75 per week, how long will it take me to earn \$1260?
11. If an automobile carriage runs an average of 125 miles per day, how long will it be in running 1500 miles?
12. If an ocean steamer averages 375 miles per day, how long will it be in going from New York to Liverpool, 3000 miles?
13. What is the value of a man's taxable property if he pays \$12 $\frac{1}{2}$ on each 25 dollars' worth and in all the sum of \$125?

- 14.** If a horse can on an average trot 1 mile in $5\frac{1}{8}$ minutes, how far will he trot in 94 minutes?
- 15.** If $2\frac{5}{8}$ pounds of butter cost \$1.00, what will 210 pounds cost?
- 16.** If $\frac{1}{5}$ of my daily income is \$2.62 $\frac{1}{2}$, what is my income for 30 days?
- 17.** If I buy porkers at \$3.87 $\frac{1}{2}$ per head and sell them at \$4.62 $\frac{1}{2}$, how many must I sell to gain in all \$24?
- 18.** If \$42.25 is the divisor and \$126 $\frac{3}{4}$ is the dividend, what is the quotient?
- 19.** $24\frac{3}{4}$ cubic feet (of stone) equal 1 perch. Find the number of perches in $74\frac{1}{4}$ cubic feet.
- 20.** How many pounds of sugar must be sold at $5\frac{1}{2}$ cents a pound to realize \$22?
- 21.** Shoes are sold at \$2.62 $\frac{1}{2}$ per pair. How many pairs can be bought for 63 dollars?
- 22.** Some land was sold at \$31.25 an acre. How many acres were bought for \$2000?
- 23.** $16\frac{1}{2}$ feet equal 1 rod. How many rods are there in 990 feet?
- 24.** What is the value of:
1. $3150 \div 13\frac{1}{8}$?
 4. $\$800 \div 16.66\frac{2}{3}$?
 7. $\$15.00 \div .18\frac{3}{4}$?
 2. $4650 \div 19\frac{3}{8}$?
 5. $\$370 \div 23.12\frac{1}{2}$?
 8. $\$56.00 \div .01\frac{1}{4}$?
 3. $4550 \div 81\frac{1}{4}$?
 6. $\$87.50 \div 17.50$?
 9. $\$231 \div 12\frac{5}{6}$?
- 25.** At $18\frac{3}{4}$ cents per pound, how many pounds of beef can be bought for \$9?
- 26.** If I buy land at \$20.50 per acre, and sell it at \$31.75 per acre, how many acres must I sell to gain \$900?
- 27.** How much tape, at \$.01 $\frac{1}{4}$ per yard, can be bought for 63 cents?

LESSON XLIII.

Partition.

1. Two men jointly earned \$144. One worked 22 days and the other 14 days. Their wages per day being the same, how much of the total sum should each receive?

2. George and Joseph expended 96 cents for confec-tions, the former paying 10 cents as often as the latter paid 6 cents. How much money did each expend?

3. In how many hours will two men meet, and how far will each have travelled, if they are now 240 miles apart, and approach each other at the rate of 7 miles and 5 miles per hour respectively?

4. Divide 144 into two parts that shall be to each other as 5 to 4.

SUGGESTION.— $5 + 4 = 9$; $\frac{5}{9}$ of 144 = 1 part.

5. If the cost of boarding 25 horses is \$50, how much must each of two men pay if one owns 13 of the horses and the other 12?

6. A. and B. built a wall for \$60. A. worked 14 days and B. 10 days. How much of the money should each receive?

7. A certain class in a school has as many boys as girls. If the teacher asked 108 questions and gave each pupil 4 questions, how many pupils were in the class? How many pupils if she gave each boy 3 questions and each girl only 1?

8. Two boys hired a tandem bicycle for \$11. One used it 8 days and the other 14 days. How much did each have to pay?

9. Two men hired a boat for \$42. One man used it 32 days and the other 52 days. How much did each man pay?

10. Paul and Richard rent a pasture for \$14.40. If Paul pastures 15 cows for 5 days and Richard pastures 23 cows for 3 days, how much should each pay?

11. A man paid \$26 for the pasture of 4 horses and 6 cows. If 2 cows ate as much as 3 horses, how much did he pay for the cows and how much for the horses?

12. Divide \$60 between two men in the proportion of 7 to 8.

13. Two contractors agree to do a piece of work for \$216. One puts on 10 men and the other 21 boys. If 3 boys can do as much work as 2 men, how much should each contractor receive?

14. A. and B. conjointly undertook a piece of work. A. sent 7 men for 4 days and B. 6 men for 6 days; on the completion of the work, A. and B. received \$128. What was the share of each?

15. A man divided some apples among three boys in the proportion of 1, 2, and 3. If the first boy received 12 apples, how many did each of the others receive? How many did all receive?

16. Divide 4800 among three persons so that their shares shall be in the proportion of 7, 8, and 9.

17. A pasture was hired for the season by 3 men, A., B., and C. They agreed to pay \$84 for its use. A pastured 3 cows, B. 4 cows, and C. 5 cows. What part of the \$84 should each pay?

18. Divide \$1728 among three men in the proportion of $\frac{1}{2}$, 1, and $1\frac{1}{2}$.

19. A. and B. pay \$49 for the rent of a field; A. puts in 20 horses for 12 weeks and B. 25 horses for 10 weeks. Divide the rent fairly between them.
20. Three men have a capital of \$16,000. B.'s equals $\frac{1}{2}$ of A.'s, and C.'s is 3 times B.'s. What is the capital of each?
21. Divide 351 into three parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.
22. Gunpowder is composed of 33 parts of nitre, 7 of charcoal, and 5 of sulphur. What weight of these components is contained in 9000 pounds of gunpowder?
23. Three persons enter into partnership. A. puts into the business £240, B. £360, and C. £120. Divide fairly among them a profit of £900.
24. Three men paid \$7500 for railroad stock, paying \$100 per share. How many shares did each receive if the first man paid \$3 as often as the second paid \$5, and the third paid \$7.
25. Three persons, A., B., and C., join in business. A. invests \$250 for 2 months, B. \$300 for 3 months, and C. \$150 for 4 months. They gain \$180. Find the share of each.

LESSON XLIV.

Equal Numbers of Things Used Under Different Conditions.

1. Lady Bountiful gave some poor folks \$6 each and had \$25 remaining. Had she given them \$8 each she would have had \$3 remaining. How many poor folks did she aid?

Solution.—The equations are: 1. $\$8 - \$6 = \$2$; $\$25 - \$3 = \$22$; $\$22 \div \$2 = 11$.

2. By the second condition each person received \$2 more and all received \$22 more; therefore, the number of persons equals the quotient of 22 divided by 2, which is 11.

2. A lady shopping bought ribbon at 10 cents a yard. Had she paid 12 cents a yard, the ribbon would have cost her 24 cents more. How many yards did she buy?

3. A gentleman gave his children \$5 each, and had \$30 remaining. Had he given them \$8 each, it would have taken all his money. Find the number of children and the amount of money.

4. A lady found that if she bought lace at 10 cents a yard she would lack 9 cents to pay for it; but if she paid 7 cents she would have 9 cents remaining. How much money had she?

5. A teacher took a trip with her pupils on a trolley-car. Having expended 20 cents on each one, she had \$3 left. Had she expended 25 cents on each, she would have had \$1.00 left. How many pupils had she?

6. A farmer bought swine at \$4 each, and had \$40 remaining. Had he paid \$5 each he would have had \$20 remaining. How many swine did he buy?

7. A farmer having purchased a number of cows at \$30 a head found that he had \$40 remaining, and that if he had bought the same number at \$40 a head, he would have needed \$20 more to pay for them. How many did he buy?

8. I wish to buy a number of books. If I buy them at 30 cents I shall have \$15 left, but if I buy them at 50

cents a volume, I shall have to borrow \$5 in order to pay for them. How many books do I wish to buy?

9. A lady distributed roses to her friends, giving 5 to each, and had 5 remaining. Had she attempted to give 7 to each, one of her friends would have received none. How many roses, and how many friends had she?

10. Two girls had an equal sum of money. One bought bananas at 8 cents apiece and had 9 cents remaining; the other bought twice as many oranges at 5 cents apiece and had 5 cents remaining. How much money had each?

LESSON XLV.

Work Problems.

1. Hannah can make a dress in $3\frac{1}{4}$ days. How much of the work can she do in 1 day? $\frac{1}{2}$ day? In $\frac{2}{3}$ day?

2. Three dress-makers, A., B., and C., can make a number of dresses in $2\frac{2}{3}$ days; B. alone can make them in 8 days; in what time can A. and C. make them?

3. If B. and C. can make the dresses in $4\frac{4}{5}$ days, in what time can A. and C. each make them?

4. If 4 men can do as much work as 6 boys, and if 7 men can do a piece of work in 8 days, how long will it take 6 men and 12 boys to do the work?

5. B. can do only $\frac{2}{3}$ as much work as A. How long will both be in doing a piece of work if A. works twice as long as B. and could do the work alone in 4 days?

6. If A. can do $\frac{3}{4}$ as much work as C., and if they together can plough a field in $3\frac{1}{2}$ days, in what time can each alone do the work?

7. A., B., and C. can dig a ditch in 6 days; A. and B. in 8 days, and B. in 12 days. How long will it take each to dig it?

8. Two men, or 3 boys, can plough an acre of land in $2\frac{1}{2}$ hours. How long will it require 3 men and 2 boys to plough it?

9. A. can do a piece of work in 10 days; A. and B. can do the same work in 7 days. In how many days can B., working alone, do the work?

10. If 8 boys or 5 men can do a piece of work in 10 days, in how many days can 8 boys and 5 men do the work?

11. A. can do $\frac{4}{11}$ of a piece of work in 6 days. With B.'s aid he finishes the work in 5 days. How long would it take B. alone to do the work?

LESSON XLVI.

Sum and Difference.

1. Find the two numbers whose sum is 36 and difference 10.

Solution.—The equations are: $\frac{36}{2} + \frac{10}{2} = 18 + 5 = 23$, the greater; $\frac{36}{2} - \frac{10}{2} = 18 - 5 = 13$, the less.

If their difference were 0 the two numbers would be each $\frac{1}{2}$ of 36, or 18; but, since their difference is 10, the $\frac{1}{2}$ of 10, or 5, must be added to 18 to produce the greater number, and subtracted from 18 to produce the less number.

2. Find the numbers whose sum and difference are as follows:

Sum.	Dif.	Sum.	Dif.	Sum.	Dif.
1. 60	20.	6. $\frac{8}{15}$	$\frac{2}{15}$.	11. \$1.00	\$.30.
2. 48	12.	7. $2\frac{2}{5}$	$\frac{4}{5}$.	12. $1\frac{1}{2}$	\$.20.
3. 56	14.	8. $5\frac{1}{3}$	$1\frac{1}{3}$.	13. \$10	\$ 4 $\frac{1}{2}$.
4. 50	12.	9. $7\frac{1}{2}$	$2\frac{1}{3}$.	14. \$5.50	\$ 1 $\frac{3}{5}$.
5. 98	26.	10. 10	$2\frac{1}{2}$.	15. \$6.80	\$ 1 $\frac{4}{5}$.

3. Two men began work with the same daily wages; one of them showing greater efficiency, received 50 cents more per day; the sum of their daily pay was then \$5.00. What was the pay of each?

4. The weight of a bushel of wheat and a bushel of corn is 116 pounds; the difference of their weights is 4 pounds. What is the weight of each?

5. A watch and chain cost \$124. The watch cost \$100 more than the chain. What did each cost?

6. A telegraph-pole is 45 feet in length; the part above ground is 31 feet longer than the part under ground. Find the length of each part.

7. A man had two equal flocks of sheep; after selling 20 from one flock and 30 from the other he had 70 left. How many sheep had he at first in each flock?

8. Two men each bought a horse at the same price. One sold his for \$12 less than cost and the other sold his for \$10 less than cost. The sum received for both was \$90. How much did the men pay for their horses?

9. Find the number whose $\frac{3}{4}$ increased by 12 and the sum multiplied by 5 gives $\frac{3}{4}$ of 600.

10. Find the number whose $\frac{4}{5}$ multiplied by 6 gives a product whose $\frac{2}{3}$ is 2000.

11. Two collectors, A. and B., received in one day

\$327.00. Find how much each received, if A. received \$27 more than $\frac{1}{2}$ of what B. received.

12. Find the ages of two men, knowing that one is 10 years more than twice as old as the other, and that the difference of their ages is 32 years.

13. A lady bought a bicycle, and after paying for it \$25 more than $\frac{1}{2}$ of her money, found that she had \$100 left. How much did the bicycle cost her?

14. A fisherman sold 25 shad less than $\frac{3}{4}$ of his whole number, and selling the rest at 20 cents apiece received \$10.00. How many had he at first?

15. Boy A. has $1\frac{1}{2}$ times as many marbles as boy B., but if A gives 3 of his marbles to B. they will have the same number. How many have each?

16. Henry's age is $\frac{2}{3}$ of William's age, and in 5 years the sum of their ages will be 58 years. How old is each?

17. A yacht sailed north a certain distance, and then turning sailed south 100 miles more than $\frac{1}{3}$ of the distance it sailed north. Assuming that it sailed in all 1200 miles, how far did it sail north and how far south?

18. A man and his son agreed to plough a certain field for \$24. The son ploughed $\frac{1}{3}$ of the field + 3 acres, and received \$11. How many acres did each plough?

19. A certain number $\times 6 \div 3 - 5 + 7 = 44$. What is the number?

20. If $\frac{6}{11}$ of a certain number is divided by 5, $\frac{6}{5}$ of the quotient is $3\frac{1}{2}$. What is the number?

21. If $\frac{1}{3}$ of a number less 6 equals 3, and $\frac{2}{3}$ of the number less 6 equals 12, what is the number?

22. If a number be increased by $\frac{1}{2}$ and $\frac{1}{2}$ of itself, the sum equals $\frac{1}{2}$ of $\frac{1}{2}$ of 390. What is the number?

DENOMINATE NUMBERS.

LESSON I.

Denominate Numbers are concrete numbers applied to measurements. In common use are measures of:

(1), Value; (2), Weight; (3), Length; (4), Surface; (5), Volume; (6), Capacity; (7), Time; and (8), Circular Measure.

UNITED STATES MONEY.

Table.

10 Mills (m.)	= 1 Cent (c.).
10 Cents	= 1 Dime (d.).
10 Dimes	= 1 Dollar (\$).
10 Dollars	= 1 Eagle (E.).

Exercises.

1. How many mills in 1 cent? In 3c.? 5c.? 8c.? 9c.? 10c.?
2. How many cents in 1 dime? In 4d.? 6d.? 9d.? 7d.? 10d.?
3. How many cents in $\frac{1}{2}$ dime? In $\frac{1}{4}$ d.? $\frac{1}{8}$ d.? $\frac{3}{10}$ d.? $\frac{1}{10}$ d.?
4. How many dimes in \$1? \$2? $\$2\frac{1}{2}$? \$8? $\$8\frac{3}{4}$? $\$6\frac{3}{10}$? \$10?
5. How many dollars in 1 eagle? 5 E.? 6 E.? $7\frac{1}{2}$ E.? $9\frac{3}{10}$ E.? 10 E.

6. Five eagles equal how many dollars? Dimes?
Cents? Mills?
7. Six dollars equal how many dimes? Cents?
Mills? Eagles?
8. \$3 equal how many cents? Dimes? Mills?
Eagles?
9. How many cents in $\$ \frac{1}{2}$? $\$ \frac{1}{4}$? $\$ \frac{3}{4}$? $\$ \frac{1}{8}$? $\$ \frac{5}{8}$? $\$ \frac{7}{8}$?
 $\$ \frac{8}{8}$?
10. How many eagles in \$50? \$25? 500 dimes?
6000 cents?
11. What part of \$1 is $12\frac{1}{2}$ c.? 25c.? $37\frac{1}{2}$ c.? $62\frac{1}{2}$ c.? $87\frac{1}{2}$ c.?
12. How many cents equal $\frac{1}{8}$ of a dollar? $\frac{5}{8}$ of a dollar?
 $\frac{7}{8}$ of a dollar?

ENGLISH OR STERLING MONEY.

Table.

4 Farthings (far.)	= 1 Penny (d.).
12 Pence	= 1 Shilling (s.).
20 Shillings	= 1 Pound, or Sovereign (£).

$$1 s. = 24 c.$$

$$1 d. = 2 c.$$

$$1 \text{ £} = \$4.86.$$

Exercises.

1. How many farthings are there in 1 penny? 2d.? 5d.? 8d.? 12d.?
2. How many pence are there in 8 far.? 12 far.? 20 far.? 36 far.?

3. How many pence are there in 1 shilling? 3s.? 5s.? 8s.? 10s.?
4. How many farthings are there in 1s.? 1s. 6d.? 2s. 9d.? 5s. 11d.?
5. Reduce to shillings 24d.; 60d.; 70d.; 90d.; £5; £5 6s.

Problems.

1. What is the value in United States money of £1 1s. 1d.?
 2. How many pounds will 60 bushels of wheat cost at 4s. per bushel?
 3. What will 50 lb. of sugar cost at 3d. per pound?
 4. What will 49 lb. of butter cost at 2s. per pound?
 5. If 17 sovereigns form a column one inch high, what is the value in shillings of a column 5 inches high?
 6. Find the value of 18 articles valued at £1 10s. each.
 7. What is the value of 90 articles valued at £2 16s. 6d. each?
-

LESSON II.

TROY WEIGHT.

Table.

24 Grains (gr.)	= 1 Pennyweight (pwt.).
20 Pennyweight	= 1 Ounce (oz.).
12 Ounces	= 1 Pound (lb.).

1 Troy lb.	= 5760 gr.
1 Avoirdupois lb.	= 7000 gr.
1 Troy oz.	= 480 gr.
1 Avoirdupois oz.	= 437½ gr.

Exercises.

1. How many grains are there in 2 pwt.? 3 pwt.? 5 pwt.? 10 pwt.?
2. How many grains are there in $2\frac{1}{2}$ pwt.? $3\frac{1}{2}$ pwt.? $4\frac{1}{2}$ pwt.?
3. How many ounces in 1 lb.? $3\frac{1}{2}$ lb.? $4\frac{3}{4}$ lb.? $5\frac{3}{8}$ lb.? $8\frac{1}{2}$ lb.?
4. Reduce 3 oz. to pennyweights; $4\frac{1}{2}$ oz.; $5\frac{3}{8}$ oz.; 2 lb.; 2 lb. 5 oz.
5. How many grains are there in 1 oz.? 1 lb.? 3 lb.? 3 lb. 3 oz.?

Problems.

1. If 1 pwt. of gold costs \$1, what will 2 lb. cost?
2. How much is received for $4\frac{1}{2}$ oz. of old gold material at \$.50 a pennyweight?
3. How many spoons, each weighing $1\frac{1}{2}$ ounces, can be made from a silver bar weighing 2 lb. 3 oz.?
4. A gold watch case weighing 2 oz. was sold at 2 cents a grain. How much was received for it?
5. A silver ring that weighs 10 pwt. was sold for \$1.12 $\frac{1}{2}$ per ounce. Find how much it cost the buyer.
6. At 6 cents a grain, what is the cost of a gold ring weighing 6 pwt. 12 gr.?
7. If 5 grains are used in one button, how many buttons can be made from 1 oz. 5 pwt. of gold?

AVOIRDUPOIS WEIGHT.

Avoirdupois Weight is used in weighing heavy articles, *except* gold and silver.

Table.

16 Ounces (oz.)	= 1 Pound (lb.).
100 Pounds	= 1 Hundred-weight (cwt.).
20 Hundred-weight 2000 Pounds	} = 1 Ton (T.).

2240 Pounds = 1 Long Ton.

Exercises.

1. How many ounces in 1 lb. avoirdupois? 2 lb.? 5 lb.? 10 lb.?
2. How many pounds in 48 oz.? 64 oz.? 96 oz.? 112 oz.? 144 oz.?
3. How many ounces in $1\frac{1}{2}$ lb.? $1\frac{1}{4}$ lb.? $2\frac{1}{4}$ lb.? $5\frac{1}{2}$ lb.? $9\frac{1}{4}$ lb.?
4. How many pounds in $1\frac{1}{2}$ cwt.? $1\frac{1}{4}$ cwt.? $2\frac{3}{8}$ cwt.? $5\frac{7}{8}$ cwt.?
5. How many pounds in 20 cwt. 24 lb.? In 2 long tons?

Problems.

1. Find the difference between 5 long tons of coal and 5 short tons of coal.
2. The difference just found is what fractional part of a short ton? Of a long ton?
3. Find the cost of 6 lb. 12 oz. of butter at 30 cents a pound?
4. What will 2 cwt. of coffee cost, at the rate of 3 pounds for 60 cents?
5. What must I pay for 6 spring chickens, each weighing 2 lb. 8 oz., at 20 cents a pound?

6. Explain why a troy ounce is heavier than an avoirdupois ounce?

7. How much heavier is an avoirdupois pound than a troy pound?

APOTHECARIES' WEIGHT.

Apothecaries' Weight is used in weighing medicines required for prescriptions.

Table.

20 Grains (gr.)	= 1 Scruple (sc. or ʒ).
3 Scruples	= 1 Dram (dr. or ʒ).
8 Drams	= 1 Ounce (oz. or ʒ).
12 Ounces	= 1 Pound (lb. or lb.).

Exercises.

- How many grains in $6\frac{1}{2}$? $9\frac{1}{2}$? $12\frac{1}{2}$? $7\frac{1}{2}\frac{1}{2}$?
- How many scruples in $7\frac{1}{2}$? $8\frac{1}{2}$? $6\frac{1}{2}\frac{1}{2}$? $9\frac{1}{2}\frac{1}{2}$?
- How many drams in $12\frac{1}{2}$? $51\frac{1}{2}$? $27\frac{1}{2}$? $42\frac{1}{2}$?
- How many drams in $6\frac{1}{2}$? $7\frac{1}{2}\frac{1}{2}$? $8\frac{1}{2}\frac{1}{2}$? $11\frac{1}{2}$?
- How many ounces in $96\frac{1}{2}$? $144\frac{1}{2}$? $1728\frac{1}{2}$?
- How many pounds in $84\frac{1}{2}$? $144\frac{1}{2}$? $1728\frac{1}{2}$?

Problems.

- How many 2-grain pills can be made from a dram of quinine?
- How many 4-grain pills can be made from $6\frac{1}{2}$ 2-drams of quinine?
- Find the cost of $8\frac{1}{2}$ 6-ounces of a medicinal substance at 25 cts. an ounce.

4. What is the cost of $6\frac{3}{5}$ of a drug sold at 2 cts. per scruple?
 5. What is the value of $504\frac{3}{5}$ of chloride of sodium, sold at $1\frac{1}{2}$ cts. per pound?
 6. How many 5-grain pills can be made from $73\frac{2}{3}$ of calomel?
-

LESSON III.

LINEAR MEASURE.

Linear Measure is used in measuring length.

Table.

12 Inches (in.)	= 1 Foot (ft.).
8 Feet	= 1 Yard (yd.).
$5\frac{1}{2}$ Yards } 16 $\frac{1}{2}$ Feet }	= 1 Rod (rd.).
320 Rods	= 1 Mile (mi.).

4 Inches	= 1 Hand.
6 Feet	= 1 Fathom.
3 Feet	= 1 Pace.
8 Furlongs	= 1 Mile.

Exercises.

1. How many inches are there in 1 ft.? 3 ft.? 7 ft.? 12 ft.?
2. How many inches are there in 2 ft. 9 in.? 3 yd.? $1\frac{1}{2}$ yd.?
3. How many feet are there in 36 in.? 84 in.? 132 in.?

4. How many yards are there in 10 ft.? 18 ft.? 51 ft.? 1728 in.?

5. How many yards are there in 2 rods? How many feet in 2 rods?

Problems.

1. On my bicycle yesterday I rode 5 miles. How many rods did I ride?

2. My horse is 84 in. high. How many hands high is he?

3. Compute the number of yards in a mile. The number of feet.

4. A sounding gives 40 fathoms of sea-depth. What is the depth in feet? In yards?

5. How much will 25 yd. of garden hose cost at 9 cts. a foot?

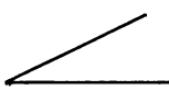
6. How many rails, 10 yd. long, will be required to build a railroad 5 miles long?

7. Find the cost of 1100 yd. of fence, at \$5 a rod.

SURFACE MEASURE.

1. **Surface or Square Measure** is used in measuring surface. A surface has only length and breadth.

This page at which you are looking is a surface.



Angle.

2. **An Angle** is the difference in direction of two lines drawn from the same point.

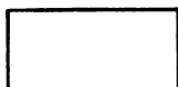
3. A **Square** has four equal sides and four equal angles. The equal angles are called *right angles*.



Square.

Table.

144 Square Inches (sq. in.)	= 1 Square Foot (sq. ft.).
9 Square Feet	= 1 Square Yard (sq. yd.).
30 $\frac{1}{2}$ Square Yards	= 1 Square Rod (sq. rd.).
160 Square Rods	= 1 Acre (A.).
640 Acres	= 1 Square Mile (sq. mi.).



Rectangle.

The name **Rectangle** applies to the square and to all other figures with four sides and four right angles.

Exercises.

1. How many square inches are there in 3 sq. ft.? 5 sq. ft.? 10 sq. ft.?
2. In 10 sq. yd., how many square feet? 12 sq. yd.? 15 sq. yd.? 20 sq. yd.?
3. In 3 sq. rd., how many square yards? 4 sq. rd.? 8 sq. rd.? 10 sq. rd.?
4. 320 acres is what part of a square mile?
5. Compute the number of square yards in an acre.

Problems.

1. If you divide a 10-acre field into lots, each of which contains 100 sq. rds., how many lots will there be?
2. How many acres in a field 60 rods long and 20 rods wide?
3. A lot 16 rods long contains $\frac{1}{2}$ an acre. How wide is it?

4. 5 boards are each 16 ft. long and 15 in. wide. How many square feet in them?

5. Draw a figure to show the difference between a surface of 4 sq. ft. and one 4 ft. square.

LESSON IV.

VOLUME MEASURE.

1. **Volume or Cubic Measure** is used in measuring that which has length, breadth, and thickness.

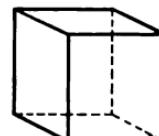
2. The volume of a body is called its **Solid Contents**, and the body is called a **Solid**.

3. A solid with six equal square faces is called a **Cube**.

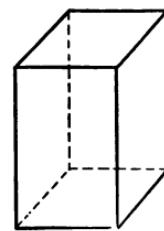
When each face is an inch square, the solid is a *cubic inch*.

When each face is a foot square, the solid is a *cubic foot*.

4. The name **Rectangular Prism** applies to cubes and other solids whose faces are rectangles.



Cube.



Prism.

Table.

1728 Cubic Inches (cu. in.)	= 1 Cubic Foot (cu. ft.).
27 Cubic Feet	= 1 Cubic Yard (cu. yd.).
128 Cubic Feet	= 1 Cord (ed.) of wood.
16 Cubic Feet	= 1 Cord Foot (ed. ft.).
24 $\frac{1}{2}$ Cubic Feet	= 1 Perch of stone.

Exercises.

1. How many cubic inches are there in 5 cu. ft.? $\frac{1}{2}$ cu. ft.? $\frac{3}{4}$ cu. ft.? 10 cu. ft.?
2. How many cubic feet are there in 3 cu. yd.? 5 cu. yd.? 9 cu. yd.? 12 cu. yd.?
3. How many cubic feet in 3 cords? $5\frac{1}{2}$ cd.? 10 cd.? 12 cd.? 15 cd.?
4. How many cubic feet in 2 perches? 3 perches? 4 perches? 5 perches?
5. 96 cu. ft. equal what part of a cord? $55\frac{1}{2}$ cu. ft.? $6\frac{5}{8}$ cu. ft.?

Problems.

1. Show that the volume of a right-prism equals the product of its length, width, and height.
2. What is the value of a pile of wood 123 ft. long, 8 ft. high, and 4 ft. wide, at \$5 a cord?
3. A cistern 20 ft. long and 15 ft. wide is full of water. How much water must be discharged to lower the surface of the water 7 ft.?
4. A brick measures 8 in. x 4 in. x 2 in. How many bricks in a pile measuring 8 ft. x 4 ft. x 2 ft.?
5. The cost of a prismatic stone 14 feet long, 10 feet wide, at \$.50 a cubic foot is \$350. Find the thickness of the stone.
6. What will it cost to dig a cellar 37 feet long, 27 feet wide, and 7 feet deep at \$.50 a cubic yard?
7. If one edge of a cubic block is 6 in., how many square inches are in its entire surface, and how many cubic inches do the faces enclose?

LESSON V.

LIQUID MEASURE.

Liquid Measure is used in measuring liquids.

Table.

4 Gills (gi.)	= 1 Pint (pt.).
2 Pints	= 1 Quart (qt.).
4 Quarts	= 1 Gallon (gal.).

$31\frac{1}{2}$ Gallons = 1 Barrel (bbl.).
 63 Gallons = 1 Hogshead (hhd.).
 1 Gallon = 231 Cubic Inches.

Exercises.

- How many gills are there in 2 pt.? 5 pt.? 3 qt. 1 pt.? $1\frac{1}{2}$ pt.?
- How many quarts are there in 3 gal.? $3\frac{3}{4}$ gal.? 8 gal.? $11\frac{1}{2}$ gal.?
- How many pints are there in 2 gal. 3 qt.? 6 gal. 2 qt. 1 pt.?
- How many gallons are there in 8 qt.? 15 qt.? 32 pt.? 56 pt.?
- What part of a gallon is 4 gills? 3 qt. 3 gi.? 3 qt. 1 pt.? 1 pt. 1 gi.?

Problems.

- What is the gain on milk, per quart, when it is bought at 16 cents a gallon and sold at 4 cents a pint?
- What is the gain on 40 gallons of molasses bought for \$30 and sold at 30 cts. a quart?

3. How many times can a vessel holding $2\frac{1}{2}$ qt. be filled from a vessel holding 10 gallons?
4. What is the cost of 4 barrels of kerosene bought at 5 cts. per gallon?
5. I have a can 11 in. long, 9 in. wide, and 7 in. deep. How many gallons of wine does it hold?

DRY MEASURE.

Dry Measure is used in measuring grain, fruit, and vegetables.

Table.

2 Pints (pt.).	= 1 Quart (qt.).
8 Quarts	= 1 Peck (pk.).
4 Pecks	= 1 Bushel (bu.).

1 Bushel = 2150.42 Cubic Inches.

Exercises.

1. In 1 peck there are how many quarts? Pints? In 1 bu. how many quarts?
2. In $\frac{1}{2}$ bu. how many cubic inches? In 1 pk.? In 1 qt.?
3. Describe a cubic inch. A 2-in. cube contains how many cubic inches?
4. In 100 quarts how many pecks? How many bushels?
5. How many pints in a bushel? How many bushels in 192 pints?

Problems.

1. A boy bought a bushel of chestnuts for \$2.40, and sold them at 9 cts. a quart. What did he gain?
2. If I pay \$1.28 for a bushel of blackberries, how much do I pay per quart?

3. When apples sell at 20 cts. a peck, how much are they worth a bushel?
 4. I paid \$1.60 per bushel for peanuts, and sold them at 5 cents per $\frac{1}{2}$ pint. How much did I gain on each peck?
 5. At \$1.00 per bushel, find the cost of 4 bu. 3 pk. 2 qt. of apples.
 6. How many cubic inches in 10 bushels of wheat?
-

LESSON VI.

DIVISIONS OF TIME.

Table.

60 Seconds (sec.)	= 1 Minute (min.).
60 Minutes	= 1 Hour (hr.).
24 Hours	= 1 Day (da.).
7 Days	= 1 Week (wk.).
12 Months	= 1 Year (yr.).
365 Days	= 1 Year (yr.).
366 Days	= 1 Leap Year (l. yr.).
100 Years	= 1 Century (C.).

Centennial years exactly divisible by 400, and other years exactly divisible by 4, are leap years.

Table.

1. January (Jan.)	= 31 da.	7. July (July)	= 31 da.
2. February (Feb.)	= 28 or 29* da.	8. August (Aug.)	= 31 da.
3. March (Mar.)	= 31 da.	9. September (Sept.)	= 30 da.
4. April (Apr.)	= 30 da.	10. October (Oct.)	= 31 da.
5. May (May)	= 31 da.	11. November (Nov.)	= 30 da.
6. June (June)	= 30 da.	12. December (Dec.)	= 31 da.

*One day added to make leap year.

A Useful Rhyme.

Thirty days hath September,
April, June, and November.
All the rest have thirty-one,
Excepting February, which stands alone
With twenty-eight, till one day more
We add to it one year in four.

Exercises.

1. How many days are there in 5 weeks? 15 wk.? 50 wk.? 52 wk.?
2. 52 weeks lack how many days of making a common year?
3. How many days make a leap year?
4. How many minutes in a day? How many seconds in $\frac{1}{4}$ day?
5. Name the months that have each 30 days. How many days in summer?

Problems.

1. Which of the following years are leap years: 1700? 1760? 1776? 1800? 1876? 1880? 1890? 1894? 1900? 2000?
2. How many more days are there in summer than in winter?
3. What date is 87 days before June 6th?
4. What is the date 50 days after Mar. 7th?
5. If a common year begins on Sunday, on what day of the week does it end? How does a leap year end that begins on Sunday?
6. The length of a day equals twice the time at which the sun sets. How long is the day when the sun sets at 10 minutes before 5 o'clock?

LESSON VII.

COUNTING.

The following denominations are frequently used in counting:

Table.

12 Things = 1 Dozen (doz.).
12 Dozen = 1 Gross (gr.).
12 Gross = 1 Great Gross (G. gr.).
20 Things = 1 Score.

Stationers' Table.

24 Sheets = 1 Quire (qr.).
20 Quires = 1 Ream (R.).
2 Reams = 1 Bundle.
5 Bundles = 1 Bale.

Exercises.

1. How many dozen in 2 gross? In $\frac{1}{2}$ gross? In $\frac{1}{2}$ a great gross?
2. How many sheets in 4 quires? In $2\frac{1}{2}$ quires? In $\frac{1}{2}$ a ream?
3. How many things in 4 score? In 3 score and 10? In $3\frac{1}{2}$ score?
4. How many quires in 5 bundles? How many sheets?
5. How many single things in a great gross? How many dozen?

Problems.

1. What will 6 gross of pens cost, if the wholesale rate is 2 pens for a cent?
2. What cost $\frac{1}{2}$ gross of lead pencils at 25 cents per dozen?
3. If a single garment requires 6 buttons, a great gross will supply how many like garments?
4. How much must be paid for a ream of paper at $\frac{1}{2}$ a cent per sheet?
5. I bought buttons for 25 cents per gross, sold them at 10 cents per dozen, and gained \$3.50. How many gross did I sell?

LESSON VIII.**Miscellaneous Problems.**

1. What will 50 bushels of wheat cost at 75 cents a bushel?
2. What will 12 dozen eggs cost at $16\frac{2}{3}$ cts. a dozen?
3. How old will you be when you have reached "3 score and 10"?
4. What will 1000 spools of thread cost at $6\frac{1}{2}$ cts. a spool?
5. What will 300 yd. of muslin cost at $33\frac{1}{3}$ cts. a yard?
6. How much will a ream of paper cost at 10 cts. a quire?
7. What cost 2 oz. of gold, if 3 pwt. cost \$2.70?
8. I paid \$1.50 for a ream of paper and sold it at 20 cts. a quire. How much did I gain?
9. What will 3 quarts of kerosene cost at 40 cents a gallon?

10. If 15 miles, on a bicycle, require 1 hr. 45 min., how long will 300 miles require?
11. How many feet high is a horse measuring 16 hands?
12. At 12 cents a peck, how many bushels of apples can be bought for \$6.00?
13. Assume that you have paid 40 cents for a gross of pens, and that you will sell them at 1 cent apiece. How much will you gain by the sale?
14. At what price must $\frac{1}{3}$ dozen chairs, worth \$15.00 a dozen, be sold in order to gain 50 cents apiece?
15. How many rods in 66 feet? How many yards?
16. If 25 lb. of flour cost \$1.25, what will 3 cwt. cost?
17. At \$2.40 per gross, how much must be paid for 84 lead pencils?
18. How many baskets, holding $2\frac{1}{2}$ pecks each, will 5 bushels of apples fill?
19. If you travel by railroad 1000 miles per day, what is your average rate per hour?
20. At 10 cents a square yard, what will it cost to paint a ceiling 20 feet long and 18 feet wide?
21. How much will a piece of ribbon 2 yd. 2 ft. 6 in. long cost at 12 cts. per yard?
22. How many rods of fence will enclose a farm $\frac{1}{4}$ mile square?
23. What is the weight of 12 loads of hay, each weighing 1 T. 3 cwt. 50 lb.?
24. How many cubic yards in a rectangular ditch 3 ft. wide, 3 ft. deep, and 21 ft. long?
25. How many yards of carpeting, 1 yard wide, will cover a floor 20 ft. long and 21 ft. wide?

LESSON IX.

1. How many cups, holding $\frac{1}{2}$ pt. each, can be filled from a coffee urn holding 2 gal. 3 qt. 1 pt.?
2. How many balls, each weighing 6 lb. $10\frac{2}{3}$ oz., can be made from a ton of iron?
3. If two men build 176 rd. of fence in $5\frac{1}{2}$ days, how much do they average per day?
4. How many perches of masonry in a wall 99 ft. long, 5 ft. high, and 2 ft. thick?
5. Find the cubic contents of a rectangular solid measuring as follows: Length, 2 yd.; breadth, 1 yd. 1 ft.; depth, 1 yd.
6. A cubic foot of water weighs about 1000 oz. Find the weight of water in a cistern measuring 12 ft. by 6 ft. by 4 ft.
7. A motor car ran from John O'Groats to Land's End, a distance of 918 miles, in 18 days. What was the average rate per day?
8. A cellar is 30 ft. long and 20 ft. wide. At what depth was 50 cu. yd. of earth removed?
9. A cable that weighs 1760 pounds per mile weighs how much per foot?
10. How many hands high is a horse that is 5 ft. 3 in. high?
11. What is the cost of 1 bu. 3 pk. 4 qt. of potatoes at 64 cents a bushel?
12. If a sloth can advance only 100 yd. in a day, in how many days can it advance a mile?
13. How many yards of fence will be required to enclose a farm of 80 acres, if the farm is 160 rods long?

14. How many dollars are there in £1 10s. 6d.?
 15. What is the cost, at \$24 per ounce, of a gold ring weighing 4 pwt. 12 gr.?
 16. The distance from New York to Philadelphia is 90 miles. What is the rate per hour of a train that runs this distance in 2 hr. 15 min.?
 17. How many quarts are there in a barrel that contains $2\frac{1}{2}$ bushels?
 18. How many square yards of carpet in a piece that is 56 yards long and $\frac{3}{4}$ yard wide?
 19. What is the value, at \$85 an acre, of a square field whose side is 80 rods?
 20. A cubic foot of water weighs $62\frac{1}{2}$ lb. How many cubic inches weigh an ounce? A cubic inch weighs how many ounces?
 21. A rectangular piece of land 1 mile long and $2\frac{1}{2}$ rods wide contains how many acres?
 22. There are 26 days for labor in a month. Find the value of a day's labor, if a man earns \$66 in 1 mo. 20 da.
 23. George Washington was born February 22, 1732. If he were living to-day, what would be his age?
 24. How often is 15 inches contained in 60 yards?
 25. How many minutes are there in the month of November?
-

LESSON X.

1. Find the cost of 6 boards, each 15 in. wide and 20 feet long, at 5 cents a square foot?
2. If 37 sheep produce 1 cwt. of wool, how much do 185 sheep produce?

3. Find the value of 24 things at £1 5s. each.
4. If a ton of coal costs £1 6s. 8d., what will 10 tons cost in U. S. money?
5. If $4\frac{1}{2}$ barrels of flour cost \$32.30, what will $7\frac{1}{2}$ barrels cost?
6. A tea-dealer bought 25 lb. of tea for \$10. He sold $\frac{1}{2}$ of it for $1\frac{3}{4}$ times what it all cost? What was his selling price per pound?
7. A pond of water measures 4 fathoms 3 feet 8 inches in depth. How many inches deep is it?
8. How many silver spoons can be made from 4 oz. of silver, and how much silver will remain, if each spoon weighs 8 pwt. 7 gr.?
9. If a watch gained 1 min. 15 sec. daily, how much did it gain during February, 1899?
10. How many rods of fence will enclose a farm $\frac{1}{2}$ of a mile square?
11. I bought 10 bushels of chestnuts at \$3.50 a bushel, and sold them at 5 cents a pint? What was my loss?
12. If 10 casks hold 308 gal. 3 qt. 1 pt., how much does one of the casks hold?
13. A cubic foot of air weighs $1\frac{1}{4}$ ounces. How many pounds of air does a room contain, which is 16 feet long, 14 feet wide, and 10 feet high?
14. If 1 can of milk contains 8 gal. 2 qt. 1 pt. of milk, 5 such cans contain how much milk?
15. A merchant owning $\frac{1}{4}$ of a vessel, sold $\frac{2}{3}$ of his share for 904 dollars. What was the value of the vessel?
16. Allowing 22 bricks to each cubic foot of wall, find the number of bricks required to build a wall 100 yd. long, 11 ft. high, and 1 ft. thick?

17. A pile of wood having the same width and height is 20 feet long and contains 10 cords. What is the width and height of the pile?
18. How many 8-in. cubes can be cut from a block of wood 2 ft. long, 1 ft. wide, and 1 ft. thick?
19. What will be the cost of a pile of wood, 36 feet long, 6 feet high, and 4 feet wide, at \$.50 per *cord foot*?
20. 6 equal packages of medicine contain 13 lb. 7 $\frac{1}{2}$ 23 1 $\frac{1}{2}$ 4 gr. How much medicine is in each package?
21. What cost 2 bu. of berries at 5 cts. per quart?
22. What will it cost to fence a garden 10 rd. long and 6 rd. wide at \$1 a rod?
23. A rectangular field containing 1 acre is 40 rd. long. How many rods around the field?
24. What will 1 bu. 1 pk. of chestnuts cost at 8 cts. per quart?
25. A man put 5 gal. 2 qt. of syrup into bottles holding 2 qt. each. How many bottles did it require?
26. How long is the day when the sun rises at 20 minutes to 5 and sets 21 minutes after 7?
27. If a man spends 50 cts. a day during April, May, and June, what does he spend in the three months?
28. A girl had a piece of cardboard 5 in. \times 7 in. She bought a piece twice as large. Give the dimensions of the new piece.
29. Find the entire surface of a cube whose edge measures 15 inches.
30. Gunpowder was invented in the year 1356. How many years ago?
31. How many $2\frac{1}{2}$ -ft. steps will a man take in walking 10 miles?

PERCENTAGE.

LESSON I.

1. Percentage means computation *by the hundred*, and has 100 for its unit. One per cent. of any number is $\frac{1}{100}$ of it; 5 per cent. is $\frac{5}{100}$ of it.

Per cent. is a contraction of the Latin *per centum*, by the hundred.

2. The result of computation is also called Percentage.

$\frac{1}{100}$ of \$500 = \$25, the percentage.

3. The symbol for per cent. is %. Per cent., however, may be expressed in 5 different ways: 6 per cent. = 6% = .06 = $\frac{6}{100}$ = $\frac{3}{50}$.

The symbol form, or the common fractional form, is best suited to the purposes of Mental Arithmetic.

4. Give the following symbol forms their simplest fractional form: 20% ($20\% = \frac{20}{100} = \frac{1}{5}$), 5%, 25%, 50%, 80%, 75%, 100%, 15%, 35%, 45%, 55%, 95%.

5. Change the following symbol forms to their simplest fractional form:

$$1. 12\frac{1}{2}\%. [12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{25}{200} = \frac{1}{8}.]$$

$$2. 37\frac{1}{2}\%, 62\frac{1}{2}\%, 87\frac{1}{2}\%, 6\frac{1}{4}\%, 33\frac{1}{3}\%, 66\frac{2}{3}\%, 16\frac{2}{3}\%.$$

The Percentage.

1. What is 60% of 120?

Solution.—1. The process is $60\% = \frac{3}{5}$; $\frac{3}{5}$ of 120 = 72.

2. $60\% = \frac{60}{100} = \frac{6}{10} = \frac{3}{5}$; $\frac{3}{5} = 3$ times $\frac{1}{5}$; $\frac{1}{5}$ of 120 = 24, and $\frac{3}{5}$ of 120 = 3 times 24, or 72.

3. Therefore, 60% of 120 is 72.

NOTE.—The second step may be omitted.

2. What is 30% of 90? 50% of 250? 25% of 84?
2% of 150? 5% of 80? 40% of 75? 15% of 300?

3. What is 25% of \$500? 6% of \$500? $12\frac{1}{2}\%$ of \$96?
 $33\frac{1}{2}\%$ of 24? $37\frac{1}{2}\%$ of \$96? $62\frac{1}{2}\%$ of \$64?

4. What is $\frac{3}{8}\%$ of \$1200?

Solution.—1. The process is $1\% = \frac{1}{100}$; $\frac{3}{8}\% = \frac{3}{8} \times \frac{1}{100} = \frac{3}{800}$; $\frac{3}{800}$ of 1200 = 8.

2. Therefore, $\frac{3}{8}\%$ of \$1200 = \$8.

5. How much is $\frac{3}{4}\%$ of 24? $\frac{1}{2}\%$ of \$600? $\frac{1}{4}\%$ of \$500?

6. How much is $\frac{5}{8}\%$ of \$320? $\frac{3}{5}\%$ of \$1000; $\frac{3}{8}\%$ of
\$5100?

7. How much is $1\frac{1}{4}\%$ of \$800? $1\frac{1}{2}\%$ of \$7200? $3\frac{1}{2}\%$ of
of \$210?

8. Find $\frac{5}{8}\%$ of \$2400; $1\frac{7}{8}\%$ of \$160; $2\frac{1}{8}\%$ of \$1800;
 $2\frac{1}{8}\%$ of \$1600.

Problems.

1. A man invested \$960 in merchandise; made sale of the goods and gained 25%. Find the amount of his gain.

2. I paid \$600 for a house, and sold it at a gain of $33\frac{1}{3}\%$. How much did I gain?

3. If \$500 were deposited in a bank and 16% of it drawn out, how many dollars were drawn out?

4. Wheat bought at \$1.00 per bushel was sold at a loss of 30%. What was received for it per bushel?

5. A farmer, having 75 sheep, sold $66\frac{2}{3}\%$ of them. How many did he retain?

6. Find the percentage on \$6000 at 1%.
 7. Find the percentage on \$6000 at $\frac{1}{4}\%$.
 8. Find the reduction on a bill of \$24 at $\frac{1}{2}\%$.
 9. In order to secure \$2500, a man paid $\frac{1}{8}\%$ of that sum to a collector. How much did the collector receive?
-

LESSON II.

The Rate.

1. Two-thirds of anything denotes what per cent. of it?

Solution.—1. Anything = $\frac{100}{1}$, or 100% of itself; hence, $\frac{2}{3}$ of anything = $\frac{2}{3}$ of 100%, or $66\frac{2}{3}\%$ of itself.

2. Therefore, $\frac{2}{3}$ of anything = $66\frac{2}{3}\%$ of it.

2. What per cent. of a number is $\frac{1}{2}$ of it? $\frac{1}{3}$? $\frac{1}{5}$? $\frac{3}{5}$? $\frac{3}{4}$? $\frac{2}{7}$? $\frac{3}{8}$? $\frac{2}{9}$? $\frac{5}{4}$? $\frac{9}{10}$?

3. What per cent. of a man's salary is denoted by $\frac{1}{6}$ of it? $\frac{1}{8}$? $\frac{1}{25}$? $\frac{1}{4}$? $\frac{2}{3}$? $\frac{7}{8}$? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{1}{250}$? $\frac{1}{300}$?

4. Express as rate per cent.: $\frac{13}{20}$, $\frac{4}{25}$, $\frac{9}{40}$, $2\frac{1}{4}$, $3\frac{1}{2}$, $7\frac{1}{6}$.

5. If a lady expends $\frac{1}{6}$ of her money for lace and the rest for silk, what per cent. of the money did each cost?

6. I bought a bill of goods and for cash I received $\frac{1}{20}$ of the bill off, and $\frac{1}{10}$ of the remainder off. What rates per cent. were deducted?

7. I planted a field in corn, cabbage, and potatoes. The corn occupied $\frac{2}{5}$ of the field; $\frac{1}{4}$ of the remainder was given to the cabbage, and the rest to the potatoes. Find the rate per cent. of space given to each crop.

8. A flock of sheep is pastured in 4 lots. The first lot contains $\frac{1}{4}$ of them; the second lot $\frac{1}{6}$; the third lot

$\frac{2}{3}$; and the fourth lot the remainder. What per cent. of them is in the fourth lot?

9. What per cent. of 5 is 4?

Solution.—1. 4 is $\frac{4}{5}$ of 5; $\frac{4}{5} = \frac{80}{100}$ or 80%.

2. Therefore, 4 is 80% of 5.

10. What part and what per cent. of 8 is 5? Of 24 is 8? Of 24 is 3? Of 30 is 6? Of 10 is 2? Of 48 is 36?

11. What part and what per cent. of 50 is 30? Of 50 is 15? Of 70 is 50? Of 25 is 15? Of 45 is 35? Of 100 is 75? Of $37\frac{1}{2}$ is $12\frac{1}{2}$?

12. What per cent. of 4 is 1? Of 2 is 4? Of 12 is 18? Of 20 is 28? Of 16 is 2? Of 2 is 6? Of 1 is 3?

13. What per cent. of $\frac{4}{3}$ is $\frac{2}{3}$?

Solution.—1. $\frac{4}{3} = \frac{12}{9}$; $\frac{2}{3} = \frac{6}{9}$; 10 is $\frac{10}{9}$, or $\frac{5}{9}$ of 12; $\frac{5}{9} = 83\frac{1}{3}\%$.

2. Therefore, $\frac{2}{3}$ is $83\frac{1}{3}\%$ of $\frac{4}{3}$.

14. What per cent. of $\frac{4}{5}$ is $\frac{1}{2}$? Of $\frac{5}{6}$ is $\frac{3}{5}$? Of $\frac{4}{5}$ is $\frac{2}{3}$? Of $\frac{4}{5}$ is $\frac{2}{3}$? Of $\frac{6}{7}$ is $\frac{1}{2}$? Of $\frac{5}{6}$ is $\frac{1}{2}$? Of $\frac{5}{6}$ is $\frac{2}{3}$? Of $\frac{5}{6}$ is $\frac{3}{5}$? Of $\frac{5}{6}$ is $\frac{4}{5}$?

15. What per cent. of $1\frac{1}{2}$ is $\frac{1}{2}$? Of $2\frac{1}{2}$ is $1\frac{1}{2}$? Of $4\frac{1}{2}$ is $2\frac{1}{2}$? Of $1\frac{1}{2}$ is $1\frac{1}{3}$? Of 5 is $3\frac{1}{3}$? Of $5\frac{1}{4}$ is $1\frac{1}{3}$?

LESSON III.

Loss and Gain.

The percentage of **Gain** or **Loss** is invariably reckoned on the cost.

1. At a gain or loss of 1, 2, 3, 4, or 5%, what part of the cost equals the gain or loss?

2. At a gain or loss of 10, 11, 12, 13, or 15%, what part of the cost equals the gain or loss?

3. If a suit of clothing was bought for \$20 and sold for \$30, what was the gain per cent.?

SUGGESTION.— $\frac{1}{2} = \frac{1}{2} = 50\%$.

4. Hannah bought a dress for \$5, and sold it for \$7. What was her gain per cent?

5. Victoria bought a coat for \$25 and sold it for \$20. What was the rate per cent. of loss?

6. A drover bought six pairs of oxen for \$450, and afterwards sold each ox for \$40. What was his gain per cent.?

7. Edna sold her bicycle for \$75, which was $\frac{1}{2}$ of what she paid for it. What per cent. did she lose?

8. A farmer who had 40 sheep sold 15. What per cent. did he sell? What per cent. did he retain?

9. I had 150 acres of land. I sold 30 acres. What per cent. of my land did I sell? What per cent. did I retain?

10. A hogshead contains 63 gallons. 21 gallons of the contents of a hogshead having been withdrawn, what per cent. remains?

11. An apple vender, having bought apples at 4 for 3 cents, sold them at 4 for 5 cents. What was his gain per cent.?

12. A farmer, having raised 280 bushels of grain, sold A. 70 bushels, sold B. $\frac{1}{2}$ of the remainder, and $\frac{2}{3}$ of what still remained to C. What per cent. of the whole had he left?

13. What per cent. of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}$ is $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{5}{6}$?

LESSON IV.

The Cost or Base.

1. Find the sum of which \$20 is 25%.

Solution.—1. $25\% = \frac{1}{4}$; $\frac{1}{4}$ the sum = \$20; $\frac{4}{4}$ the sum = \$80.

2. Therefore, \$20 is 25% of \$80.

2. 13% of my farm is 52 acres. How many acres does my farm contain?

Solution.—1. 100% of my farm is the whole of it.

2. 13% of it = 52 acres; $1\% = 4$ acres; $100\% = 400$ acres.

3. Therefore, my farm contains 400 acres.

3. Find the number of which 30 is $16\frac{2}{3}\%$.

4. On hay sold at \$12 per ton there was a profit of $33\frac{1}{3}\%$. How much did the hay cost?

5. I sold my carriage for \$300, and by so doing lost 25%. How much did the vehicle cost me?

6. My expenses were \$600, which was $62\frac{1}{2}$ per cent. of my income. What was my income?

7. Mr. Brook sold a horse for \$225, which was 90% of what he paid for it. What did the horse cost him?

8. A. sold a house to B. at a gain of 25%, and B. sold it to C. for \$224, which was at a gain of 12%. How much did A. pay for the house?

9. Duncan lost 50% on his watch by selling it for \$50. If he had sold it at a gain of 40%, what would he have received for it?

10. A merchant invests \$750 in merchandise, which was 15% of his entire capital. What was the amount of his capital?

11. Mr. Smith, owning $\frac{1}{4}$ of a ship, sold 30% of his share for \$1200. What was the value of the ship?

12. I sold cloth at $\$2\frac{1}{2}$ per yard and lost 16 $\frac{2}{3}\%$. In order to gain 16 $\frac{2}{3}\%$, for how much should I have sold it?

13. A man sells a horse for \$351 at a profit of 17 per cent. What did the horse cost him?

14. If sugar sold at 9 cents per pound gives a profit of 14%, what was the cost price per cwt.?

15. A tradesman makes 35 per cent. profit on his outlay. What did he pay for goods that he sells for \$324?

16. A dealer sold a piano 40% below the advertised price and yet made a profit of 25%. If the advertised price was \$500, find how much the piano cost the dealer.

17. A lady spent 37 $\frac{1}{2}\%$ of her money, and then found that it would take 80% of the remainder to buy a dress for \$30. How much had she at first?

18. A man sold two houses for \$3600 each; on one he gained 20%, and on the other he lost 20%. What was his gain or loss by the transaction?

19. Two shares of stock were sold for \$99 each; on the first there was a loss of 10%; on the second a gain of 10%. What was gained or lost by the transaction?

20. A. and B. invested in a business equal capital. A. gained 21% and B. 31%. Assuming that B. gained \$1440 more than A., find the capital of each.

21. An army lost 25% of its men in battle, and 33 $\frac{1}{3}\%$ of the remainder were discharged. How many men were in the army at first, if there were 5000 men remaining?

LESSON V.

The Percentage as Commission, Brokerage, etc.

When an agent sells, his commission is a percentage of his sales.

When an agent buys, his commission is a percentage of his purchase.

An agent may be a commission merchant, a broker, a collector, a lawyer, a bank, or any person or institution acting for an employer or principal.

1. An agent makes a sale of \$900 worth of goods, and receives 5% for his services. What is the amount of his commission?

2. A collector, whose commission was $2\frac{1}{2}\%$, made collections amounting to \$3000. How much should he pay over to his principal?

3. Dr. Green paid $12\frac{1}{2}\%$ for the collection of \$9600. How much did he pay?

4. A real estate agent sold a house and lot for \$5360. If his commission is 5%, how much should he receive for his services?

5. An auctioneer sells a sewing-machine for \$30, and receives $2\frac{1}{2}\%$ for his services. How many dollars does the owner receive?

6. I send an agent money enough to buy \$150 worth of shoes at a commission of 4%. How much money do I send him?

7. If a lawyer exacts 25% for his services, what will he realize from securing a claim of \$4936?

8. An attorney collects a claim of \$600; he pays \$25 costs and charges 5% for collection. What does he pay the owner of the claim?

9. A man employs a broker to purchase for him 20 shares of railroad stock whose face value is \$100 per share. If the broker charges $\frac{1}{4}\%$, what is his commission?

10. A bank in St. Louis charged $\frac{1}{4}\%$ for collecting a draft on Philadelphia for \$1500. How much did the bank deduct for its services?

11. A broker purchases £20,000 worth of goods. What is his commission at $\frac{1}{4}\%$?

12. A travelling salesman's commission was 10 per cent. of his sales. If his sales averaged \$1250 per month last year, how much commission did he receive?

LESSON VI.

The Base.

The **Base** in commission is the amount of the *sale, purchase, or collection*.

1. If I sell goods at 4% commission and receive \$60, what amount have I sold?

Solution.—1. 4% of the amount = \$60; 1% of the amt. = \$15; 100% of the amt. = \$1500.

2. Therefore, the amount required is \$1500.

2. An agent's commission, at $1\frac{1}{2}\%$, for selling a house was \$150. For how much money was the house sold?

3. A man gained \$20 by selling a boat for 20% above

its value. What would he have gained by selling it for 15% above its value?

4. An agent made a sale, and, after deducting from the proceeds his commission at 5%, returned to his employer \$28,500. Find the amount of his commission?

Solution.—1. He returned $100\% - 5\% = 95\%$ of the sale.

$$2. \ 95\% = \$28,500; \ 1\% = \$300; \ 5\% = \$1500.$$

3. Therefore, his commission was \$1500.

5. \$187.50 was a broker's commission at $\frac{1}{8}\%$ for selling stock. What was the par value of the stock he sold?

6. \$1050, sent to an agent, includes both the investment he is to make and his commission at 5%. Find the amount of the investment?

Solution.—1. 100% , the investment, + 5%, the commission, = 105%.

$$2. \ 105\% = \$1050; \ 1\% = \$10; \ 100\% = \$1000.$$

3. Therefore, the agent invested \$1000.

7. What amount of goods can I purchase for \$520, if my agent charges me 4% on the money expended?

8. I have received \$432 to buy goods, and am to retain 8% of the money expended. How much of the money can I expend?

9. When a miller is taking 10% toll, how much grain must be taken to mill in order to bring away the flour of 3 bushels?

10. If I send an agent \$630 to purchase goods for me, and he charges 5% commission, what sum does he expend for goods?

LESSON VII.

Insurance.

Insurance is of two kinds, **Property Insurance** and **Personal Insurance**.

The contract made is called the **Policy**.

The sum paid for insurance is called the **Premium**.

1. A store worth \$20,000 was insured at $1\frac{1}{4}\%$. What was the cost of insurance or premium?

Solution.—1. $1\frac{1}{4}\% = \frac{5}{40} = \frac{1}{8}$; $\frac{1}{8}$ of \$20,000 = \$250.

2. Therefore, the cost of insurance = \$250.

2. What premium must be paid for an insurance of \$4000 on a house at $1\frac{1}{4}\%$?

3. At 2% premium, what will be the cost of a policy of insurance for a ship worth \$50,000?

4. A house was insured for $\frac{1}{4}$ of its value at $2\frac{1}{4}\%$, and the premium was \$66. What was the value of the house?

5. If you insure your goods for \$6000, paying a premium of $2\frac{3}{5}\%$, what does the insurance cost you?

6. What will be the cost of an insurance policy for \$5500 at $2\frac{1}{2}\%$ premium?

7. What will it cost to insure a building worth \$9600 for $\frac{3}{4}$ of its value, at $1\frac{1}{2}\%$?

8. A dealer buys 1500 bushels of wheat at \$1.00 per bushel, and insures it at \$1.10 per bushel at $2\frac{1}{2}\%$. If the wheat is destroyed by fire, what is his gain?

9. I paid a premium of \$80 for an insurance of \$6400 on a house. What rate did I pay?

SUGGESTION.—\$80 = $\frac{1}{8}$ of \$6400; $\frac{1}{8}$ of 100% = $1\frac{1}{4}\%$.

10. The premium for insuring a house for $\frac{5}{6}$ of its value at $\frac{4}{5}\%$ was \$39 per year. What was the amount of the policy?

11. A ship owner paid \$690 for insuring $\frac{2}{3}$ the value of a ship, which was \$69,000. What was the rate of insurance?

12. What must a property worth \$1950 be insured for, at $2\frac{1}{2}\%$, so as to include the premium in case of loss?

SUGGESTION.— $100\% - 2\frac{1}{2}\% = 97\frac{1}{2}\% = \$1950.$

13. If A. and B. own a property worth \$15,000, how much of the insurance premium at $1\frac{1}{2}\%$ should each pay, if $\frac{2}{3}$ of A.'s share of the property equals $\frac{2}{3}$ of B.'s?

14. Find how many partners were in the firm, if each had invested \$2000, and if for insurance, at $1\frac{1}{2}\%$ on their whole capital, they paid a premium of \$120?

LESSON VIII.

Interest.

Interest is money paid for the use of money, and depends both upon a certain rate per cent. and the length of time the money is in use.

The money used is the **Principal (Base)**.

The interest for one year is the **Percentage**.

The interest for a longer or a shorter time than one year is the product of the percentage and the time expressed in years or in the fraction of a year.

Exercises.

1. What is the interest of \$400 for 1 year at 6%?

Solution.—1. The interest for 1 year = $\frac{6}{100}$, or $\frac{3}{50}$ of the principal.

$$2. \frac{3}{50} \text{ of } 400 = \$\frac{1200}{50} = \$24.$$

3. Therefore, the interest of \$400 for 1 year = \$24.

2. What would be the interest on the above sum for 2 yr.? $2\frac{1}{2}$ yr.? 5 yr.?

3. Find the interest of:

- | | |
|----------------------------|--|
| 1. \$500 for 1 yr. at 5%. | 11. \$900 for 2 yr. at 2%. |
| 2. \$900 for 1 yr. at 4%. | 12. \$250 for 3 yr. at $2\frac{1}{2}\%$. |
| 3. \$700 for 1 yr. at 10%. | 13. \$500 for 4 yr. at 3%. |
| 4. \$400 for 1 yr. at 9%. | 14. \$350 for 5 yr. at $3\frac{1}{2}\%$. |
| 5. \$600 for 1 yr. at 6%. | 15. \$400 for $1\frac{1}{2}$ yr. at 4%. |
| 6. \$300 for 1 yr. at 8%. | 16. \$850 for $2\frac{1}{2}$ yr. at $4\frac{1}{2}\%$. |
| 7. \$200 for 1 yr. at 3%. | 17. \$275 for $3\frac{1}{2}$ yr. at 5%. |
| 8. \$1000 for 1 yr. at 1%. | 18. \$450 for $1\frac{1}{2}$ yr. at $5\frac{1}{2}\%$. |
| 9. \$250 for 1 yr. at 7%. | 19. \$50 for 1 yr. at 6%. |
| 10. \$850 for 1 yr. at 5%. | 20. \$25 for $\frac{1}{2}$ yr. at 60%. |

Problems.

1. In computing interest, a year is reckoned 12 months, and a month 30 days.

2. A month, therefore, is considered $\frac{1}{12}$ of a year, and a day $\frac{1}{30}$ of a month, or $\frac{1}{360}$ of a year.

3. What is the interest of \$6000 at 6% for 3 yr. 3 mo. 20 da.?

Solution.—1. 20 da. = $\frac{2}{3}$ mo.; $3\frac{2}{3}$ mo. = $\frac{11}{3}$ mo. = $\frac{11}{36}$ yr.

2. $3\frac{11}{36}$ yr. = $\frac{119}{36}$ yr. = the whole time.

3. $\frac{6}{100}$ or $\frac{3}{50}$ of the prin. = the int. for 1 yr.

4. $\frac{11}{36}$ of $\frac{3}{50}$, or $\frac{11}{600}$ of the prin. = the int. for the whole time.

5. $\frac{11}{600}$ of \$6000 = \$1190, the interest required.

6. Therefore, the interest of \$6000 for 3 yr. 3 mo. 20 da. = \$1190.

Hence, you perceive the importance of acquiring facility (1) in reducing the given time to the fraction of a year, and (2) in finding the fraction of the principal that equals the interest.

Exercises.

1. What part of the principal equals the interest in :

1. 1 yr. 3 mo. at 6% ? 6. 8 yr. 2 mo. at 1% ?

2. 2 yr. 6 mo. at 5% ? 7. 10 yr. 3 mo. at 2% ?

3. 3 yr. 4 mo. at 7% ? 8. 9 yr. 9 mo. at 3% ?

4. 4 yr. 8 mo. at 8% ? 9. 6 yr. 1 mo. at 5% ?

5. 5 yr. 2 mo. at 4% ? 10. 7 yr. 5 mo. at 6% ?

2. What part of the principal equals the interest in :

1. 1 yr. 1 mo. 10 da. at 6% ?

Solution.—1. $10 \text{ da.} = \frac{1}{3} \text{ mo.}; 1\frac{1}{3} \text{ mo.} = \frac{4}{3} \text{ mo.} = \frac{4}{36} \text{ yr. or } \frac{1}{9} \text{ yr.}$ Or, $1 \text{ mo. } 10 \text{ da.} = 40 \text{ da.} = \frac{40}{360} = \frac{1}{9} \text{ yr.}$

2. $1\frac{1}{3} \text{ yr.} = \frac{10}{9} \text{ yr.} =$ the whole time given.

3. $\frac{6}{100}$ or $\frac{3}{50}$ of the prin. = the int. for 1 yr.

4. $\frac{10}{9}$ of $\frac{3}{50}$, or $\frac{1}{5}$ of the prin. = int. for whole time.

5. Therefore, $\frac{1}{15}$ of the prin. = the int. required.

2. 2 yr. 2 mo. 20 da. at 6% ? 6. 5 yr. 5 mo. 10 da. at 7% ?

3. 1 yr. 6 mo. 15 da. at 5% ? 7. 2 yr. 6 mo. 12 da. at 8% ?

4. 3 yr. 3 mo. 12 da. at 4% ? 8. 1 yr. 3 mo. 27 da. at 6% ?

5. 4 yr. 4 mo. 5 da. at 3% ? 9. 3 yr. 4 mo. 24 da. at 6% ?

3. Find the interest of \$300, at 6% for 2 yr. 2 mo. 12 da. For 5 yr. 4 mo. 24 da. at 5% ?

4. Find the interest of \$900, using 9% as the rate, and 1 yr. 10 mo. 20 da. as the time.
5. Find the interest of \$500, $7\frac{1}{2}\%$ as the rate; and 2 yr. 3 mo. 18 da. as the time.
6. Find the interest of \$750, if the rate is 8%, and the time 5 yr. 3 mo. 27 da.
7. What is the interest of \$600 for 30 days at 6%?

Solution.—1. At 6%, $\frac{3}{50}$ of the prin. = the int. for 360 da.

$$2. \text{ 30 da.} = \frac{1}{12} \text{ of } 360 \text{ da.}$$

$$3. \frac{1}{12} \text{ of } \frac{3}{50}, \text{ or } \frac{1}{200} \text{ of the prin.} = \text{the int. for 30 da.}$$

$$4. \frac{1}{200} \text{ of } \$600 = \$3, \text{ the interest required.}$$

Or, we may say, "The int. for 60 da. at 6% = $\frac{1}{100}$ of the prin.; the int. for 30 da. = $\frac{1}{2}$ of $\frac{1}{100}$ or $\frac{1}{200}$ of prin.; $\frac{1}{200}$ of \$600 = \$3.00."

8. What is the interest of:

- | | |
|-----------------------------|-------------------------------|
| 1. \$4375 for 60 da. at 6%? | 6. \$4800 for 30 da. at 5%? |
| 2. \$1800 for 30 da. at 6%? | 7. \$3300 for 90 da. at 8%? |
| 3. \$2500 for 20 da. at 6%? | 8. \$1400 for 60 da. at 7%? |
| 4. \$3000 for 90 da. at 4%? | 9. \$1500 for 15 da. at 6%? |
| 5. \$2000 for 30 da. at 9%? | 10. \$18.40 for 90 da. at 6%? |
-

LESSON IX.

The Amount = the Principal + the Interest.

1. What is the amount of \$1200 for 1 yr. 3 mo., at 6%?

Solution.—1. At 6%, the int. = $\frac{3}{50}$ of the prin., and the amount = $\frac{43}{50}$ of the principal.

$$2. \frac{43}{50} \text{ of } \$1200 = \$1290, \text{ the amount required.}$$

2. What is the amount of:

1. \$1800 for 6 yr. at 6%?
2. \$240 for 5 yr. at 5%?
3. \$400 for 6 yr. 6 mo. at 6%?
4. \$2000 for 5 yr. 6 mo. at 6%?
5. \$3200 for 3 yr. 3 mo. at 7%?
6. \$1500 for 2 yr. 2 mo. at 8%?
7. \$400 for 1 yr. 5 mo. at 6%?
8. \$500 for 2 yr. 8 mo. at 6%?
9. \$3000 for 2 yr. 2 mo. 15 da. at 8%?
10. \$200 for 1 yr. 2 mo. 3 da. at 12%?
11. \$120 for 30 da. at 6%?
12. \$450 for 60 da. at 7%?
13. \$3000 for 15 da. at 4%?
14. \$2400 for 45 da. at 2%?
15. \$1600 for 90 da. at 5%?

3. A. and B. have inherited the amount of \$1200 on interest for 5 yr. at 6%. Divide the amount so that A. shall have $\frac{2}{3}$ as much as B.

4. Find the amount of a note for \$1500 for 3 mo. at 10%.
5. Find the amount of C.'s and D.'s money in 4 years at 6%, assuming that C. has \$300, which is $\frac{2}{3}$ the amount of D.'s money.
6. The amount of E.'s money for 2 yr. 6 mo. at 6% is \$250 less than the amount of F.'s money at 8%. If E.'s money equals F.'s, how much has each?
7. A. and B. together have \$100, and $\frac{2}{3}$ of A.'s money is equal to $\frac{9}{10}$ of B.'s money. Find the amount of each man's sum, if put at interest for 2 yr. 6 mo. at 6%.

8. A 60-day note for \$1000, with interest at 6%, was paid at maturity. What was the amount due?

LESSON X.

To Find the Principal.

1. What principal will give \$30 yearly interest at 6%?

Solution.—1. $\frac{6}{100}$ or $\frac{3}{50}$ of the principal = 1 year's interest.

2. $\frac{3}{50}$ of the prin. = \$30; $\frac{1}{50}$ of the prin. = \$10; $\frac{50}{50}$ of the prin. = \$500.

3. Therefore, \$500 is the principal required.

2. What principal will give \$2.88 in 8 mo. at $4\frac{1}{2}\%$?

Solution.—1. 8 mo. = $\frac{2}{3}$ yr.; $4\frac{1}{2}\% = \frac{9}{200}$; $\frac{2}{3}$ of $\frac{9}{200}$ = $\frac{3}{100}$.
2. $\frac{3}{100}$ of prin. = \$2.88; $\frac{100}{3}$ of prin. = \$96.

3. Find the principal in the following cases:

Rate.	Time.	Interest.
1. 5% ; 4 yr. 6 mo.;		\$180.
2. 4% ; 3 yr. 9 mo.;		\$510.
3. 6% ; 4 yr. 6 mo. 24 da.;	\$137.	
4. 4% ; 2 yr. 9 mo.;		\$13.20.
5. 8% ; 3 yr. 3 mo.;		\$156.

4. A certain principal produces \$240 interest at 6%. What would be the interest if the rate were 4%?

5. The interest of $\frac{2}{3}$ of A.'s money for 6 yr. 3 mo., at 4%, is \$150. What is A.'s money?

6. What principal will in 10 yr. 10 mo. 10 da., at 12%, give \$782?

7. My brother and I, at the same time, put at interest equal sums of money, he obtaining 8% and I but 6%. At the end of the year he received \$40 more interest than I received. What was our principal?

8. The amount of Crawford's money for 6 yr. at 6% is \$300 more than the amount of it for 3 yr. at 10%. How much money had he?

9. The wages paid a man and a boy will, in 1 yr. 9 mo., at 6%, yield \$42 interest. What wages were paid each, if $\frac{1}{2}$ of what the man received equaled $\frac{3}{4}$ of what the boy received?

10. A man bought 100 horses and gave in payment his note bearing interest at 6%. In 2 mo. 15 da., when he paid the note, the interest due was \$62.50. What was the average cost of a horse?

LESSON XI.

To Find the Principal, the Amount being Given.

1. What principal will amount to \$39.60 in 2 yr. 6 mo. at 4%?

Solution.—1. 6 mo. = $\frac{1}{2}$ yr.; $2\frac{1}{2}$ yr. = $\frac{5}{2}$ yr.

2. At 4%, $\frac{1}{25}$ of the prin. = the int. for 1 yr., and $\frac{5}{2}$ of $\frac{1}{25}$, or $\frac{1}{5}$ of the prin. = the int. for the given time.

3. $\frac{1}{10}$ of the prin. + $\frac{1}{5}$ of the prin. = the amount.

4. $\frac{1}{10}$ of the prin. = \$39.60; $\frac{1}{5}$ of the prin. = \$3.60; $\frac{1}{10}$ of the prin. = \$36.

5. Therefore, \$36 is the principal required.

2. What principal will amount to \$196 in 2 yr. 2 mo. 20 da. at 4%?

3. What sum in 90 days, at 6%, will amount to \$406.00?

4. What sum in 60 days, at 4%, will amount to \$3020?

5. The amount due on a note which had been on interest for 2 yr. 2 mo., at 6%, is \$565. What was the face of the note?

6. I wish to put some money on interest to benefit a needy friend. If I wish him to receive \$30 per month, how much money, at 10%, must I put on interest for him?

7. Three times A.'s money, added to $1\frac{1}{2}$ times B.'s, being on interest for 8 yr. at 12%, amounts to \$4900. How much has each, if $1\frac{1}{2}$ times B.'s money equals A.'s?

8. If $\frac{5}{6}$ of my money equals $\frac{1}{2}$ of your money, and the total sum of our money being put at interest amounts to \$11,400 in 2 yr. 4 mo. at 6%, how much have we each?

9. $\frac{1}{2}$ of Sanford's coins, plus $\frac{3}{5}$ of his bills, being put on interest for 5 yr. at 7%, amounts to \$2700. What is the value of each kind of money, if the bills were worth $2\frac{1}{2}$ times as much as the coins?

10. $\frac{2}{3}$ of A.'s money and $\frac{3}{4}$ of B.'s are to each other as 4 is to 3. How much money has each, if the sum total of their money amounts to \$26,900, at 6%, in 5 yr. 9 mo.?

LESSON XII.

To Find the Time.

1. In what time will \$2400, at 5%, yield \$144?

Solution.—1. In 1 yr. $\frac{1}{20}$ of the prin. = the int.

2. $\frac{1}{20}$ of \$2400 = \$120, the int. for 1 yr.

3. To yield \$144, the prin. will require $\frac{144}{120}$ yr. = $\frac{6}{5}$ yr. = $1\frac{1}{5}$ yr.

4. Therefore, in $1\frac{1}{5}$ yr. the given prin. will yield \$144.

2. In what time will \$9000, at 4%, yield \$540?
 3. In what time will \$72 yield \$1.44 interest, at 6%?
 4. In what time will \$45 produce \$.45 interest, at $4\frac{1}{2}\%$?
 5. If the principal is \$360, the rate 3%, and the interest \$43.20, what is the time?
 6. In what time, at 8%, will \$90 yield \$36?
 7. In what time will \$300, at 7%, amount to \$510?
 8. I gave my note for \$1500 at 6%; when I paid it the interest had amounted to \$270. Find the time.
 9. In what time will a principal double itself at 6%? At 7%? At 8%? At 10%? At 12%?
 10. A certain principal, for a certain time, at 5%, amounts to \$250, and at 8% amounts to \$280. Find the principal and also the time?
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LESSON XIII.

To Find the Rate.

1. At what rate per cent. will \$2000, in $2\frac{1}{2}$ yr., yield \$300 interest?

Solution.—1. At 1%, \$2000, in $2\frac{1}{2}$ yr., yields \$50 int.

2. To yield \$300 int. in the same time, the rate must be as many times 1% as \$50 is contained times in \$300, or 6 times.

3. Therefore, the rate required is 6%.

2. At what rate will \$38 in 2 yr. amount to \$40.28?

Solution.—1. The int. = \$40.28 — \$38.00 = \$2.28.

2. At 1%, the int. of \$38 for 2 yr. = \$.76.

3. To yield \$2.28, the rate = $\$2.28 \div \$.76 = 3$.

4. Therefore, the rate required is 3%.

3. What is the rate, if the principal is \$120; the time 1 yr. 2 mo. 15 da.; and the interest \$4.35?
4. At what rate will \$200 produce \$16 interest in 2 yr.?
5. At what rate will \$64 produce \$.64 interest in 80 days?
6. At what rate will \$80 produce \$2.40 interest in 6 mo.?
7. At what rate will \$600 in 5 yr. yield \$135 interest?
8. At what rate per cent. will \$1500 in 2 yr. 7 mo. 6 da. yield \$390?
9. At what rate per cent. will \$800 in 2 yr. 3 mo. 15 da. produce \$220 interest?
10. At what rate per cent. will \$600 in 4 yr. 6 mo. amount to \$708?
11. At what rate per cent. will a given principal gain 3 times itself in 10 years?

Solution.--To gain 3 times itself any principal must gain 300%. If it gains 300% in 10 years, in one year it will gain $\frac{1}{10}$ of 300%, or 30%. Therefore, at 30% a principal will gain 3 times itself in 10 years.

12. At what rate will a principal double itself in 12 yr.?
13. At what rate per cent. will \$400 make \$37.50, simple interest, in 1 yr. 3 mo.?
14. At what rate per cent. will \$12,000 produce \$258 interest in 4 mo. 9 da.?
15. At what rate will any sum, in 3 yr. 1 mo. 15 da., produce a sum equal to $\frac{1}{2}$ of the capital?
16. The amount of a certain principal for 4 yr. at a certain rate is \$620, and for 7 years at the same rate it is \$710. Find the rate and the principal.

LESSON XIV.

REVIEW.

1. A man buys a horse for \$250 and sells it at a profit of 20%. How much does he gain?
2. A green-grocer bought 50 bushels of potatoes at 50 cents a bushel. He gave away 20% of them in charity, and sold the remainder at a gain of 20% on the entire cost. How much did he receive per bushel?
3. Two horses were sold for \$63 each. On one the gain was $12\frac{1}{2}\%$; on the other the loss was $12\frac{1}{2}\%$. What was gained or lost by the transaction?
4. I sold my watch for \$90 and gained $33\frac{1}{3}\%$. At a gain of 40%, what would I have received for it?
5. At a commercial discount of 25 off and 20 off from the remainder, what per cent. discount do I receive?
6. A carriage was offered for sale; the builder's selling price was \$500. It was sold for cash at a discount of 20 and 5 off. How much was received for it?
7. A merchant marked an article \$1.50, and sold it at a discount of 20%. If the article cost him 80 cts., what was his gain per cent.?
8. If a man checked out 50% of his money in bank for a farm and 20% of the remainder for horses and cattle, the former sum being \$500 larger than the latter, how large was each sum?
9. A. bought stocks 20% below par, and sold them 20% above par. If he gained \$90, how many dollars' worth of stocks did he buy?
10. At what rate per cent. will a principal double itself in 16 yr. 8 mo.?

11. The amount of a certain principal at a certain rate for 6 yr. is \$1704, and for 10 yr. \$2040. Find the principal and the rate.

12. Find the rate of gain or loss when $\frac{1}{4}$ of a barrel of sugar is sold for $\frac{1}{2}$ of its cost?

13. I have paid one creditor 20% of my yearly income, and another creditor 10% of the remainder. Find my income, assuming that the first creditor received \$720 more than the other.

14. Corn and oats are produced on a farm in such quantities that 20% of the corn equals $37\frac{1}{2}\%$ of the oats. Of both there are 2300 bushels. How many bushels of each were raised?

15. 40% of A.'s money is in bank, 30% of it is loaned, the remainder, \$540, is at hand. How much money has A.?

16. A carriage-builder sold a phaeton to B. at a gain of 25%. B. sold the phaeton to C. for \$150, and thereby lost 25%. What had the phaeton cost the carriage-builder?

17. I sold my piano and received for it such a price that 30% of it equaled $37\frac{1}{2}\%$ of the cost. What per cent. did I gain or lose?

18. A dealer wished to ask for a horse such a price that after discounting it 10%, and also the remainder 10%, he might get \$162 for the horse. What was his asking price?

19. How must an article that cost \$16 be marked so that after discounting the price 20% it may be sold at a profit of 25%?

20. If you constantly sell 4 pounds of food for what 5 pounds cost, what per cent. do you gain?

LESSON XV.

1. A farmer bought sheep at \$4 a head. He sold them at a gain of $16\frac{2}{3}\%$, and made a profit of \$36. How many sheep did he buy?

2. A consignment of goods was sold for \$160 less than cost, and the loss was 20%. To gain $12\frac{1}{2}\%$, for how much must it have been sold?

3. A lady bought two kinds of tea, paying 50 and 60 cents per pound. Had the grocer mixed them and sold the mixture as a superior "blend" for 20% more profit, what would have been the cost?

4. A cashier has received in all 800 pieces of coin. The number of dimes is 30% of the number of nickels, and the number of quarter-dollars is $33\frac{1}{3}\%$ of the number of dimes. There are 100 dollar and half-dollar pieces. How many quarters, dimes, and nickels are there?

5. 15% of one-half a certain number is 6 less than $33\frac{1}{3}\%$ of $\frac{2}{3}$ of it. What is the number?

6. If in a mixture of two grades of coffee $37\frac{1}{2}\%$ of it cost 40 cents a pound and the remainder 30 cents a pound, how must the mixture be sold per pound to gain $33\frac{1}{3}\%$?

7. A girl stringing beads lost 20% of them; finding 25% of as many as were left she had 100. How many had she at first?

8. If wheat is bought for 90% of the market price and sold at a gain of 30%, at what per cent. above the market price is it sold?

9. A. and B. form a partnership in business, A. furnishing 70% of the money invested and B. the rest. In

a year, at a gain of 20%, their capital has become \$2400. How much did each gain?

10. A man after expending \$100 more than 35% of his money had \$160 left. How much had he at first?

11. I sold certain goods by the yard at a gain of 30%; but making a mistake in measuring my gain was reduced 16 $\frac{2}{3}$ %. How many inches did I sell for a yard?

12. My house was worth \$10,000; I paid \$60 for insuring it at $\frac{4}{5}\%$. For how much was my house insured?

13. Eight apples were bought for 16 cents and $\frac{1}{4}$ of them were lost. For how much must the remainder be sold to compensate for the loss?

14. The sum of 10% of a number and 5% of half the remainder is what per cent. of the number?

15. A mechanic's shop is valued at \$1900. For what sum must he have it insured, at 5%, so that in case of loss he may receive both the value of the shop and the premium?

16. I buy my goods at 25% from the list price and sell at the list price. Find my gain per cent.

17. With how much money must an agent be supplied so that he may buy 1500 lbs. of coffee, at 20 cents a pound, and retain his commission at 2 $\frac{1}{2}\%$ for buying?

18. A collecting agent whose commission was 2% remits to his employer \$2940. What amount did he collect?

19. A mowing machine was marked 33 $\frac{1}{3}\%$ above cost. It was sold for 12 $\frac{1}{2}\%$ less than marked, and the gain was \$21. How high was it marked?

20. How far above cost may a man mark his goods so that he may take off 10% therefrom, and still make 17%?

LESSON XVI.

1. Assume that as agent you received \$5121 with which to buy goods at 2% commission, after deducting \$21 for freight. Find the value of the goods bought.
2. If 6% of my income is \$300, and my income is 10% of my capital, what is my capital?
3. Mix water and wine in the proportion of 5 gallons of water to 30 gallons of wine, and find what per cent. each ingredient is of the mixture.
4. I bought a house for \$8000; money is worth 6%, and I am paying 3% for taxes and insurance. What does the house cost me as my home?
5. Property valued at \$12,000 is insured for $\frac{2}{3}$ its value at the rate of $\frac{1}{4}\%$ a year. Find the premium.
6. A note, after being on interest for 1 yr. 3 mo. 15 da. at 6%, was paid in full with a check for \$8620. What was the face of the note?
7. A man loans $\frac{1}{2}$ of his money at 6%, $\frac{1}{3}$ at 4%, and the remainder at $4\frac{1}{2}\%$; his income therefrom is \$305. Find how much money he loans?
8. Find the face of a 60-day note which, when discounted at a Philadelphia bank, at 6%, will yield \$297?
9. I buy oranges at the rate of 15 cents a dozen, and sell them at the rate of 3 for 10 cents. Find the gain per cent.
10. A lady's suit cost \$190. The making cost $33\frac{1}{3}\%$ less than the stuff, and the incidental expenses cost $33\frac{1}{3}\%$ less than the making. How much did each cost?
11. A bookseller sells books at 25% profit, but allows teachers a discount of 10%. If he receives \$27 from a teacher in payment of a bill, what is his profit?

12. A. and B. each invest \$1600 in business. At the end of the year A. has gained $2\frac{1}{4}\%$ and B. has lost $2\frac{3}{4}\%$. How much has each?

13. A bookseller buys a book whose catalogue price is \$3.50 at a discount of 20% and 5%, and sells it at 10% above the catalogue price. What per cent. profit does he make?

14. $\frac{3}{4}$ is $\frac{4}{5}\%$ of how many times $\frac{5}{8}$?

15. An agent remits to me \$950, after retaining his commission of 5%. What sum did he collect? What was the amount of his commission?

16. A merchant bought goods to the amount of \$9000. He sold them and, after deducting 10% for bad debts, realized a profit of 25%. How much did he sell them for?

17. A. sells a horse to B., gaining 20%, and B. sells it to C. for \$150, and gains 25%. What did the horse cost A.?

18. A. lost 4% of his money, and with $62\frac{1}{2}\%$ of the remainder bought three horses at \$350 each. How much money did he lose?

19. A certain principal in 3 yr. 2 mo. 18 da. at 6% amounts to \$1193. In how many years will it amount to \$1386?

20. The money of A., B., and C. placed at interest for 1 year and 6 months at 6%, amounts to \$1308. How many dollars has each, if A. has $1\frac{1}{3}$ times as much as B., and C. has $1\frac{1}{4}$ times as much as A.?

21. If the selling price is $33\frac{1}{3}\%$ above cost, name the best discount that can be given without loss?

22. If $\frac{1}{5}$ of the price received for an article equals the loss, what is the loss per cent.?

LESSON XVII.

Miscellaneous Problems.

1. If a girl should read $\frac{1}{3}$ of a book one day, $\frac{1}{6}$ of it the second day, and $\frac{1}{9}$ of it the third day, what part of the book would she read in the 3 days?
2. If by increasing $\frac{1}{2}$ of a boy's money by $\frac{1}{3}$ of his money the sum is \$4.20, how much money has he?
3. What number must be added to $13\frac{1}{4}$ to make $26\frac{1}{4}$?
4. A man sold $\frac{1}{2}$ of his goods at one time and $\frac{3}{4}$ of the remainder at another time; he had left \$500 worth. How many dollars' worth had he at first?
5. What must I pay for a gross of pencils at $\$1\frac{1}{2}$ apiece?
6. A field was ploughed in $\frac{1}{6}$ of a week; how much of that field could be ploughed in $\frac{1}{3}$ of a week?
7. If a horse consumes $\frac{1}{4}$ of a ton of hay in $\frac{1}{2}$ a month, how long will $1\frac{1}{2}$ tons last him?
8. The cost of sawing a stick of wood into 3 pieces is how many times the cost of sawing a similar stick into 2 pieces?
9. If you can row down a stream at the rate of 8 miles per hour, but can return up stream only at the rate of 5 miles per hour, how swift runs the current, and what would be your rate in still water?
10. A. can dig a ditch in 5 days and B. in 6 days. In what time will they dig it working together?
11. If 4 men can do a piece of work in 9 days, how many can do it in 4 days?
12. A pound of tea lasted a man and his wife 12 weeks, and the wife alone 48 weeks. How many weeks would it last the man alone?

13. A modiste can make a dress in 5 days, but with an assistant helping she can make it in 3 days. Find the time in which the assistant could make the dress.

14. If 5 men and 9 boys can do a piece of work in 17 days, in how many days would 9 men and 12 boys do it, the work of 2 men being equal to that of 3 boys?

15. Assume that a boy can earn by his industry \$1.40 per day. At that rate how much can he earn in 10 days, if he rests $\frac{2}{3}$ of the time?

16. A clock gains 1 minute every hour. If it marks the right time at 12 m. on Monday, how far will it be astray at 4 P.M. on Tuesday?

17. A man paid \$120 for keeping 2 horses 12 weeks. What would it cost at the same rate to keep 1 horse 5 days?

18. A drover sold 40 sheep for \$210. For those of the better quality he received \$6 apiece; for the others, \$5 apiece. How many were there of each kind?

Solution.—1. The process is $\$6 \times 40 = \240 ; $\$240 - \$210 = \$30$; $\$6 - \$5 = \$1$; $\$30 \div \$1 = 30$; 40 sheep — 30 sheep = 10 sheep.

2. Had he received \$6 apiece for the 40 sheep, he would have received for all 40 times \$6 or \$240; but he received \$240 — \$210, or \$30 less, caused by the difference in price.

3. Since the difference in price was \$6 — \$5, or \$1, and the whole difference \$30, there were 30 sheep sold at the less price, \$5. At the higher price, \$6, there were sold 40 sheep — 30 sheep = 10 sheep.

4. Therefore, there were 10 sheep of the better kind and 30 sheep of the inferior kind.

NOTE.—Had the difference in price been \$2, the number of sheep sold at the less price would have been $30 \div 2$, or 15.

19. A grocer bought red and green apples, 20 bushels in all, for \$9. For the red apples he paid \$.50 per bushel and for the green apples \$.40 per bushel. How many bushels of each kind did he buy?

20. I paid \$60 for 10 tons of coal. For stove coal I paid \$6.75 a ton, and for nut coal I paid \$5.50. How many tons of each kind did I buy?

21. My son and I worked alternately for 60 days, and received \$144. If I received \$3, and my son \$1.50 per day, how many days did each of us work?

22. A man paid \$76 for 12 barrels of flour, giving \$7 for that of the best grade, and \$5 for that of inferior grade. How many barrels of each did he buy?

23. Mr. A. bought a farm containing in part 250 acres of tilled land. As often as 5 acres had been tilled 3 acres had been left untilled. How many acres were in the farm, and how many acres of untilled land?

24. Divide \$1001 among A., B., and C., giving B. twice as much as A., and C. twice as much as B.

LESSON XVIII.

1. An artisan contracts to work at the rate of \$3 a day for 30 days. At the end of that time he received \$75. How many days did he lose?

2. A laborer agreed to work 40 days at 75 cents a day and his board, but he was to pay 50 cents a day for boarding as long as he was idle. At the end he received \$25. How many days did he work?

3. A contractor in order to reduce his expenses \$60 per day discharged 15 of his men and reduced the wages

of the rest from \$3 to $\$2\frac{1}{2}$ a day. How many men had he at first?

4. Corn and oats were sold at 50 cents and 20 cents a bushel, respectively. How many bushels of each were sold if \$3 more was received for the corn than for the oats?

5. A poultreer had a certain number of fowls; he killed at one time 5 more than $\frac{1}{2}$ of them, at another time 5 more than $\frac{1}{2}$ of the remainder, and then had 15 left. How many had he at first?

Solution.—1. Since $\frac{1}{2}$ the remainder — 5 = 15, $\frac{1}{2}$ the remainder = 20; hence, the remainder = 40.

2. Since $\frac{1}{2}$ the whole number — 5 = 40, $\frac{1}{2}$ the whole number = 45; hence, the whole number = 90.

6. A farmer sold to one man 50 acres more than $\frac{1}{2}$ his farm; to another man 50 acres more than $\frac{1}{2}$ of the remainder, and then had 150 acres left. How many acres had he at first?

7. A lady spent \$10 less than $\frac{1}{4}$ of her money and had \$43 left. How much money had she at first?

8. A merchant invested \$120 more than $\frac{1}{4}$ of his money in dry-goods, and \$100 more than $\frac{1}{6}$ of the remainder in groceries, and then had \$175 remaining. How much money had he at first?

9. A bookseller sold in one month 4 less than $\frac{1}{4}$ of his school books, the next month 4 less than $\frac{1}{5}$ of the remainder; he had then 220 left. How many had he at first?

10. A baker at an early morning hour delivered 30 more loaves of bread than $\frac{3}{5}$ of all that had been ordered, and had 60 more loaves to deliver. How many loaves had been ordered?

11. A dressmaker received \$70 more than $\frac{2}{3}$ of a bill she had sent, and \$30 was still due her. What was the amount of the bill?

12. A seamstress in trimming a dress with lace used $\frac{1}{2}$ of a yard less than $\frac{4}{5}$ of the lace supplied, and had 1 yard remaining. How many yards of lace had she?

13. After reading 60 pages of a book I found there remained 60 pages more than $\frac{2}{3}$ of the whole number of pages. How many pages did the book contain?

14. A boy, having a number of problems to solve during a certain day, made solution of 10 less than $\frac{3}{4}$ of them before 12 M., 10 more than $\frac{1}{2}$ of the remainder before 3 P.M., and had 5 left. How many problems had been assigned him?

15. An explorer essayed to climb a mountain. He ascended to a point whose height was 1000 feet less than $\frac{2}{3}$ of the whole height. After resting, he ascended $\frac{7}{10}$ of the remaining height, but stopped 900 feet from the top. What was the height of the mountain?

16. What distance does a man ride in the cars at the rate of 30 miles an hour, if he can return by carriage at the rate of 6 miles per hour, travelling in all 8 hours?

Solution.—1. 30 miles an hour = 1 mi. in $\frac{1}{30}$ hr. 6 mi. an hr. = 1 mi. in $\frac{1}{6}$ hr.

2. Hence, to ride 1 mile and return requires $\frac{1}{30}$ hr. + $\frac{1}{6}$ hr. = $\frac{6}{30}$ hr., or $\frac{1}{5}$ hr.

3. Therefore, a man can go and return as many miles as $\frac{1}{5}$ hr. is contained times in 8 hr., or 40 miles.

17. How far down stream may one go at the rate of 12 miles an hour, and return at the rate of 6 miles an hour, and be gone in all 7 hours?

18. How swift is the current of a river, if a man can row down stream 12 miles an hour and up stream at the rate of 8 miles per hour?

SUGGESTION.—Twice the rate of the current = 12 mi. — 8 mi.; for the rate of the current alone makes the difference.

19. What is a man's rate of rowing in still water, if he can row down stream 10 miles an hour, and up stream 5 miles an hour?

20. If you go away from home at the rate of 12 miles an hour, and return at the rate of 6 miles an hour, how long will it take you to go 6 miles and return?

21. A boat whose speed is 10 miles an hour goes down a stream whose current is 2 miles an hour. The boat returns in 5 hours. How far did it sail?

22. A. and B. are two places. The bicycle time from B. to A. is $2\frac{1}{2}$ times as great as the time from A. to B. The round trip is made in 7 hours. How long is the time from B. to A.?

23. A boat goes up a river at the rate of 10 miles an hour, and returns at the rate of 16 miles an hour. It is absent 13 hours. How far did it run, and what was the rate of the current?

24. A man has gone 20 miles and back in 5 hours. If the returning rate was $\frac{2}{3}$ of the going rate, what was the returning rate per hour?

25. My salary is $1\frac{1}{2}$ times my expenses, and I can save \$3600 in 12 years. What is my salary?

26. At the rate of 15 problems an hour for A., and 10 problems in 30 minutes for B., in what time can they together solve 140 problems?

LESSON XIX.

1. If you buy two lengths of garden hose and pay 10 cents and 12 cents per foot respectively, how much do you pay for the hose, there being 88 feet in all, and each length of hose costing the same sum?

2. Two pieces of dress goods were bought for the same money. The cost per yard was 20 cents and 30 cents respectively. What was the total cost, there being 50 yards in all?

3. A man rode a certain distance and walked back, having been gone $9\frac{1}{2}$ hours. What was the distance he rode, if the going was at the rate of $12\frac{1}{2}$ miles per hour and the returning at the rate of $3\frac{1}{4}$ miles per hour?

4. A steam launch has the rate of 15 miles per hour down stream and the rate of 10 miles per hour up stream. When the time of returning is $3\frac{1}{2}$ hours more than the time of going, what distance has the boat gone?

5. A boy agreed to serve an employer one year for \$400 and a bicycle. At the end of 9 months he received \$280 and the bicycle. What was the value of the bicycle?

Solution.—1. He earned $\frac{3}{4}$ of \$400 + $\frac{1}{4}$ of the value of the bicycle.

2. That is, $\$300 + \frac{1}{4}$ of the value of the bicycle.

3. He received $\$300 - \280 , or $\$20$ for $\frac{1}{4}$ of the value of the bicycle.

4. Hence, the value of the bicycle was \$80.

6. A man and his wife contracted to serve a year for \$900. The wife served only 10 months. At the end of the year they received but \$840. Find the rate at which each was paid.

7. A thief is 30 steps in advance of an officer, and takes 5 steps while the officer takes 4, but 2 of the officer's steps equal 3 of the thief's. How many steps will the officer have taken before he arrests the thief, and how many steps the thief?

Solution.—1. 2 steps of the officer = 3 steps of the thief.

2. 1 step of the officer = $\frac{3}{2}$ steps of the thief.

3. 4 steps of the officer = $1\frac{1}{2}$ or 6 steps of the thief.

4. The officer in taking 4 steps gains 6 steps — 5 steps, or 1 step of the thief.

5. To gain 30 steps of the thief the officer will take 30 times 4 steps, or 120 steps.

6. The thief will have taken, when arrested, 30 times 5 steps, or 150 steps.

8. A fox is 50 leaps before a hound and takes 3 leaps while the hound takes 2, but 2 of the hound's leaps equal 4 of the fox's. In how many leaps will the hound catch the fox?

9. If 4 men and 3 boys earn \$15 per day, and 3 men and 4 boys earn \$13 per day, how much does each earn per day?

Solution.—Multiplying the first statement by 4 and the second by 3, we have:

1. 16 men + 12 boys earn \$60 per day.

2. 9 men + 12 boys earn \$39 a day.

3. Therefore, 7 men earn \$60 — \$39 per day, or \$21, and each man \$3.

4. Since 4 men + 3 boys earn \$15, three boys earn \$3, or \$1 each.

10. If 1 bushel of wheat and 2 bushels of oats weigh 124 pounds, and if 2 bushels of wheat and 1 bushel of

oats weigh 152 pounds, what is the weight of a bushel of each?

11. At what time between 4 and 5 o'clock are the hour and minute hands of a clock together?

Solution.—The dial of a clock is divided into 12 one-hour spaces. At 12 o'clock the two hands are together. The minute hand may be regarded as setting out again to overtake the hour hand. When the minute hand has returned to 12 the hour hand has moved to 1. The minute hand has therefore gained 11 spaces in 60 minutes, and will gain the remaining 1 space in $\frac{1}{11}$ of 60 minutes, or $5\frac{5}{11}$ minutes. At 4 o'clock the hands are 4 spaces apart, and the minute hand has 4 spaces to gain before overtaking the hour hand, which it will do in 4 times $5\frac{5}{11}$ minutes, or $21\frac{9}{11}$ minutes past 4 o'clock.

12. When the hands are together between 5 and 6 o'clock, what time is it?

13. At what time between 9 and 10 o'clock do the hands of a clock point in exactly opposite directions?

14. What time is it when $\frac{1}{2}$ of the time past noon equals $\frac{1}{3}$ of the time to midnight?

Solution.—1. From noon till midnight is 12 hours.

2. Since $\frac{1}{2}$ the time past noon = $\frac{1}{3}$ of the time to midnight, the time past noon = $\frac{1}{2}$ of the time to midnight.

3. The time to midnight = $\frac{3}{2}$ of the time to midnight.

4. Therefore, $\frac{3}{2}$ of the time to midnight = 12 hours, $\frac{1}{2}$ the time to midnight = 4 hours, and the time to midnight = 8 hours.

5. Hence, the time required is 4 o'clock P.M.

15. What time is it when $\frac{1}{2}$ of the time past noon equals $\frac{1}{3}$ of the time to midnight?

16. A boy made a recitation when $\frac{3}{4}$ of the time past noon equaled the time past midnight. At what time did he recite?

17. A student has 100 historical and scientific books. Of his collection 30 are scientific. How many of the historical kind must he sell so that there may be 5 on history to 3 on science?

Solution.—30 on science = 10 times 3 on science. Therefore, there should be 10 times 5, or 50, on history; but there are 70; hence, 20 historical books must be sold.

18. A man has a mixed drove of 100 sheep and cows: he has 60 sheep. How many sheep must he sell so that he shall have but 3 sheep to 4 cows?

19. A lady having a certain sum of money in her purse went shopping with a friend who had with him \$180. The lady borrowed from her friend as much money as she already had, and then both had the same sum. How much money had the lady at first?

20. Hannah is 24 years old and her sister is 10. How long was it since Hannah was 3 times as old as her sister?

Solution.—1. Hannah is 14 years older than her sister.

2. This difference always existed.

3. When Hannah was 3 times as old as her sister the difference of their ages was twice the sister's age.

4. Hence, the sister's age at the required time was $\frac{1}{2}$ of 14 yrs., or 7 yrs.

5. Since the sister's age is now 10, 3 years ago is the answer required.

21. Mrs. P. is three times as old as her son, but in 10 years she will be just twice as old as her son. How old is each?

22. A gentleman and his wife left home in an automobile carriage and travelled at the rate of 18 miles per hour. Their vehicle becoming disabled they returned home at the rate of 6 miles per hour. How far from home did they go if they were gone 4 hours?

23. The amount of a certain principal for a certain time at 5% is \$450, and at 8% it is \$470. Find the principal and time?

24. Fifty excursionists hired a boat, but 10 had no money with which to pay the *pro rata*, and in consequence the expense of the rest was increased \$.50 each. How much was paid for the use of the boat?

25. In less than 6 months steel billets have advanced from \$15 per ton to \$31 per ton. Find the rate per cent. of advancement.

LESSON XX.

1. A certain number diminished by 30 is the same as 30 diminished by one-half the number. Find the number.

2. A man buys a piece of ground agreeing to give for it a year's labor and \$100. At the end of five-sixths of the year he ceased labor and discharged his debt by paying \$200. What was the value of his labor?

3. A sum of money was divided among A., B., and C. A. and B. received \$150; A. and C. \$216; and B. and C. \$178. How much did each receive?

4. A father takes 3 steps while his son takes 5, but 2 of the father's steps are equal to 3 of the son's. How

many steps will the son require to overtake the father, who is 24 steps ahead?

5. Four times A's money added to 5 times B's money is \$1400; but 5 times A's money added to 4 times B's is \$1300. How much has each?

6. Seven pounds of tea and 5 pounds of coffee cost \$5.60, and 5 pounds of tea and 3 pounds of coffee cost \$3.80. Find the price of each.

7. If a man receives daily £1 and spends daily 15 shillings, how long will it take him to pay a debt of £20?

8. At what time between 6 and 7 o'clock are the hands of a clock together?

9. At what time between 2 and 3 o'clock are the hands of a clock at right angles?

10. A farmer sold a lot of fowls for \$17.50; chickens at 35 cents apiece, ducks at 45 cents, and turkeys at \$1.20. If there were twice as many ducks as turkeys and twice as many chickens as ducks, how many of each were there?

11. I buy goods at 25% from list price and sell them at list price; find the per cent. of gain.

12. If 2 men dig $10\frac{1}{2}$ rods of ditch in $3\frac{1}{2}$ days, how many rods will 3 men dig in 4 days?

13. What single rate of discount is the same 10%, 10%, and 10% off?

14. What is the gain per cent. when oranges are bought at 15 cents a dozen and sold at the rate of 3 for 10 cents?

15. What is the cost of a stone walk 4 rods long and 5 feet wide, at $66\frac{2}{3}$ cents a square foot?

16. School bonds bearing $4\frac{1}{2}\%$ interest are bought at 10% below par. What rate does the purchaser make?

17. If 2 men plow 15 acres in 5 days working ten hours

a day, how many acres will 3 men plow in 4 days working 8 hours a day?

18. If I buy 120 barrels of apples at \$1.50 per barrel, and after losing 40 barrels, sell the remainder so as to gain 20% on the money invested, how much do I receive for a barrel?

19. If stocks are bought at 10% below par and sold at 5% above par, what is the gain per cent?

20. How many gold rings, each weighing 4 pwt. 4 gr., can be made from 2 oz. 10 pwt. of gold?

LESSON XXI.

1. At an election 620 votes were cast for two candidates. How many votes were cast for each, if $\frac{3}{4}$ of those cast for one equaled $\frac{4}{5}$ of those cast for the other?

2. What principal will amount to \$345, in 2 yrs. 6 mo. at 6%?

3. If $\frac{3}{4}$ of an article is sold for $\frac{7}{8}$ of the cost of the article, what is the gain per cent?

4. If 9% dividend is paid on stock bought at 40% discount, what per cent. does the investment pay?

5. A merchant sends \$3075 to his agent to buy flour at \$5 per barrel, after deducting his commission of $2\frac{1}{2}\%$. How many barrels can he buy?

6. I sell goods at 10% below marked price and still make 10% profit. What per cent. above cost was the marked price?

7. At what price must 6% bonds be purchased so as to realize $7\frac{1}{2}\%$ on the investment?

8. If stock bought at 90 pays 5% interest on the investment, at what price should the same stock be bought to pay 6%.

9. A hat cost \$4. How must it be marked so that the gain may be 25% after 20% has been deducted from the marked price?

10. Received 6% dividend on stock at 25% below par, what rate of interest did the investment pay?

11. A bookseller marks his books 25% above cost, but allows teachers 10% discount. If he receives \$27 from a teacher in payment of a bill, what is his profit?

12. How would you write thirty if the scale were 5?

13. At 8% in what time will \$250 amount to \$275?

14. If by selling goods at 4 $\frac{2}{3}$ % profit \$21 is made, find the cost and the selling price.

15. What is the insurance premium on a building valued at \$2700, insured for two-thirds of its value at 3 $\frac{1}{2}$ %?

16. Find the amount \$2500, for 3 yr. 4 mo. 24 da. at 5%.

17. What is the cost of a tract of land $\frac{1}{2}$ mile long and $\frac{1}{8}$ of a mile wide at \$50 per acre?

18. A factory valued at \$48,000 is insured for $\frac{2}{3}$ of its value. What is the rate of insurance, if the premium is \$480?

19. If a grocer should sell a tub of butter at 22c per pound he would make a profit of \$3.60; but if he should sell at 16c a pound, he would lose \$1.80. How many pounds in the tub, and what was the cost per pound?

20. What is the cost of 10 shares of railroad stock at 3 $\frac{1}{2}$ % discount, if the broker charges $\frac{1}{8}$ %?

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